Name________________________ ID_________________ Section______

1-7	/42
8	/15
9	/15
10	/15
11	/15
Total	/102

MATH 253 Exam 2 Spring 2004

Sections 504-506 P. Yasskin

On the front of the Blue Book, on the Scantron and on this sheet
write your Name, your University ID, your Section and "Exam 2."
On the front of the Blue Book copy the Grading Grid shown at the right.
Enter your Multiple Choice answers on the Scantron
and CIRCLE them on this sheet.

Multiple Choice: (6 points each. No part credit.)

1. Find the equation of the plane tangent to the graph of the function $f(x,y) = x^2 y$ at the point $(3,2,18)$.

a. $z = 12x + 9y + 18$

b. $z = 9x + 12y + 18$

c. $z = 12x + 9y - 36$

d. $z = 9x + 12y - 18$

e. $z = 4x^2y - 6xy - 2x^2$

2. The equation of the plane tangent to the graph of $z = f(x,y)$ at $(1,2)$ is $z = 4 + 2(x-1) - 3(y-2)$. Use the linear approximation to estimate $f(1.2, 1.9)$.

a. 0.7

b. 3.3

c. 3.9

d. 4.1

e. 4.7

3. Find the equation of the plane tangent to the ellipsoid $\frac{x^2}{3} + \frac{y^2}{12} + \frac{z^2}{27} = 1$ at the point $(1,2,3)$.

a. $6x + 3y + 2z = 18$

b. $6x + 3y + 2z = -18$

c. $36x + 9y + 4z = 66$

d. $36x + 9y + 4z = -66$

e. $3x + 2y + z = 10$

4. If the temperature is $T(x,y,z) = \frac{xy}{z}$ and a bird is at $(x,y,z) = (3,2,1)$, in what unit vector direction should the bird fly to warm up as quick as possible?

a. $(3,2,-6)$

b. $\left(\frac{3}{7}, \frac{2}{7}, \frac{-6}{7}\right)$

c. $(2,3,-6)$

d. $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

e. $(3,2,6)$

5. If the temperature is $T(x,y,z) = \frac{xy}{z}$ and a bird is at $(x,y,z) = (3,2,1)$ flying with velocity $\vec{v} = (2,1,3)$, what is the rate of change of the temperature as seen by the bird?

a. -25

b. -11

c. 0

d. 11

e. 25

6. If $w = x^3 + y^3$ where $x = \cos(pq)$ and $y = \sin(pq)$, find $\frac{\partial w}{\partial q}$ at $(p,q) = \left(\frac{1}{6}, \pi\right)$.

NOTE: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

a. $\frac{\sqrt{3} - 3}{16}$

b. $\frac{3 - \sqrt{3}}{16}$

c. $\frac{\sqrt{3} + 3}{16}$

d. $\frac{-3 - \sqrt{3}}{16}$

e. $\frac{3 - 2\sqrt{3}}{16}$

7. Suppose $f(x,y) = x^2 y$ where $x = x(u,v)$ and $y = y(u,v)$. Find $\left.\frac{\partial f}{\partial u}\right|_{(u,v)=(3,4)}$ given that:

$$x(3,4) = 1 \qquad \left.\frac{\partial x}{\partial u}\right|_{(u,v)=(3,4)} = 5 \qquad \left.\frac{\partial x}{\partial v}\right|_{(u,v)=(3,4)} = 7$$

$$y(3,4) = 2 \qquad \left.\frac{\partial y}{\partial u}\right|_{(u,v)=(3,4)} = 6 \qquad \left.\frac{\partial y}{\partial v}\right|_{(u,v)=(3,4)} = 8$$

a. 13

b. 26

c. 52

d. 83

e. 174

Work Out: (15 points each. Part credit possible.)

Start each problem on a new page of the Blue Book. Number the problem. Show all work.

8. $(15$ points$)$ Find 3 positive numbers a, b and c, whose product is 36 for which $a+2b+3c$ is a minimum.
You MUST solve the problem by Eliminating a Variable.

9. $(15$ points$)$ Find 3 positive numbers a, b and c, whose product is 36 for which $a+2b+3c$ is a minimum.
You MUST solve the problem by the Method of Lagrange Multipliers.

10. $(15$ points$)$ Suppose x, y and z are related by the equation $yz+xz+xy=11$.

a. Use implicit differentiation to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(1,2,3)$.

b. Find the equation of the plane tangent to the surface $yz+xz+xy=11$ at the point $(1,2,3)$.

11. $(15$ points$)$ Find all critical points of the function $f(x,y)=8x^3+y^3-12xy$ and classify each as a local minimum, a local maximum or a saddle point. Be sure to say why.
NOTE: $A^3-B^3=(A-B)(A^2+AB+B^2)$