MATH 253 Spring 2004 Section 504 P. Yasskin
Maple Quiz Solutions

```
> restart:with(VecCalc):VCalias:
```

#1

```
> f:=[x,y,z] &-> (x*y^2*z^3);
```

$$f := (x, y, z) \to x\, y^2\, z^3$$

```
> g:=[x,y,z] &-> (x^2+y^2+z^2);
```

$$g := (x, y, z) \to x^2 + y^2 + z^2$$

```
> delf:=Grad(f);
```

$$delf := [(x, y, z) \to y^2\, z^3, (x, y, z) \to 2\, x\, y\, z^3, (x, y, z) \to 3\, x\, y^2\, z^2]$$

```
> delg:=Grad(g);
```

$$delg := [(x, y, z) \to 2\, x, (x, y, z) \to 2\, y, (x, y, z) \to 2\, z]$$

```
> eqs:=op(equate(delf&@[x,y,z], lambda*delg&@[x,y,z]));
```

$$eqs := y^2\, z^3 = 2\, \lambda\, x, 2\, x\, y\, z^3 = 2\, \lambda\, y, 3\, x\, y^2\, z^2 = 2\, \lambda\, z$$

```
> constr:=g(x,y,z)=9;
```

$$constr := x^2 + y^2 + z^2 = 9$$

```
> sol:=solve({eqs,constr}, {x,y,z,lambda});
```

$$sol := \{x = \text{RootOf}(\_Z^2 - 9 + y^2), y = y, z = 0, \lambda = 0\},$$
$$\{z = z, x = \text{RootOf}(\_Z^2 - 9 + z^2), y = 0, \lambda = 0\}, \{z = 3\ \text{RootOf}(2\ \_Z^2 - 1, label = \_L1),$$
$$\lambda = \frac{27}{2}\ \text{RootOf}(2\ \_Z^2 - 3)\ \text{RootOf}(2\ \_Z^2 - 1, label = \_L1), y = \text{RootOf}(\_Z^2 - 3),$$
$$x = \text{RootOf}(2\ \_Z^2 - 3)\}$$

```
> sol1:=allvalues(sol[1]);
```

$$sol1 := \{x = \sqrt{9 - y^2}, y = y, z = 0, \lambda = 0\}, \{x = -\sqrt{9 - y^2}, y = y, z = 0, \lambda = 0\}$$

```
> p1:=subs(sol1[1],[x,y,z]);
```

$$p1 := [\sqrt{9 - y^2}, y, 0]$$

```
> p2:=subs(sol1[2],[x,y,z]);
```

$$p2 := [-\sqrt{9 - y^2}, y, 0]$$

```
> sol2:=allvalues(sol[2]);
```

$$sol2 := \{z = z, x = \sqrt{9 - z^2}, y = 0, \lambda = 0\}, \{z = z, x = -\sqrt{9 - z^2}, y = 0, \lambda = 0\}$$

```
> p3:=subs(sol2[1],[x,y,z]);
```

$$p3 := [\sqrt{9 - z^2}, 0, z]$$

```
> p4:=subs(sol2[2],[x,y,z]);
```

$$p4 := [-\sqrt{9 - z^2}, 0, z]$$

```
> sol3:=allvalues(sol[3]);
```

$$sol3 := \{z = \frac{3\sqrt{2}}{2}, \lambda = \frac{27\sqrt{6}\sqrt{2}}{8}, y = \sqrt{3}, x = \frac{\sqrt{6}}{2}\},$$

$$\left\{z=-\frac{3\sqrt{2}}{2},\lambda=-\frac{27\sqrt{6}\sqrt{2}}{8},y=\sqrt{3},x=\frac{\sqrt{6}}{2}\right\},$$

$$\left\{z=\frac{3\sqrt{2}}{2},\lambda=\frac{27\sqrt{6}\sqrt{2}}{8},y=-\sqrt{3},x=\frac{\sqrt{6}}{2}\right\},$$

$$\left\{z=-\frac{3\sqrt{2}}{2},\lambda=-\frac{27\sqrt{6}\sqrt{2}}{8},y=-\sqrt{3},x=\frac{\sqrt{6}}{2}\right\},$$

$$\left\{z=\frac{3\sqrt{2}}{2},\lambda=-\frac{27\sqrt{6}\sqrt{2}}{8},y=\sqrt{3},x=-\frac{\sqrt{6}}{2}\right\},$$

$$\left\{\lambda=\frac{27\sqrt{6}\sqrt{2}}{8},z=-\frac{3\sqrt{2}}{2},y=\sqrt{3},x=-\frac{\sqrt{6}}{2}\right\},$$

$$\left\{z=\frac{3\sqrt{2}}{2},\lambda=-\frac{27\sqrt{6}\sqrt{2}}{8},y=-\sqrt{3},x=-\frac{\sqrt{6}}{2}\right\},$$

$$\left\{\lambda=\frac{27\sqrt{6}\sqrt{2}}{8},z=-\frac{3\sqrt{2}}{2},y=-\sqrt{3},x=-\frac{\sqrt{6}}{2}\right\}$$

```
> p5:=subs(sol3[1],[x,y,z]);
```

$$p5:=\left[\frac{\sqrt{6}}{2},\sqrt{3},\frac{3\sqrt{2}}{2}\right]$$

```
> p6:=subs(sol3[2],[x,y,z]);
```

$$p6:=\left[\frac{\sqrt{6}}{2},\sqrt{3},-\frac{3\sqrt{2}}{2}\right]$$

```
> p7:=subs(sol3[3],[x,y,z]);
```

$$p7:=\left[\frac{\sqrt{6}}{2},-\sqrt{3},\frac{3\sqrt{2}}{2}\right]$$

```
> p8:=subs(sol3[4],[x,y,z]);
```

$$p8:=\left[\frac{\sqrt{6}}{2},-\sqrt{3},-\frac{3\sqrt{2}}{2}\right]$$

```
> p9:=subs(sol3[5],[x,y,z]);
```

$$p9:=\left[-\frac{\sqrt{6}}{2},\sqrt{3},\frac{3\sqrt{2}}{2}\right]$$

```
> p10:=subs(sol3[6],[x,y,z]);
```

$$p10:=\left[-\frac{\sqrt{6}}{2},\sqrt{3},-\frac{3\sqrt{2}}{2}\right]$$

```
> p11:=subs(sol3[7],[x,y,z]);
```

$$p11:=\left[-\frac{\sqrt{6}}{2},-\sqrt{3},\frac{3\sqrt{2}}{2}\right]$$

```
> p12:=subs(sol3[8],[x,y,z]);
```

$$p12 := \left[-\frac{\sqrt{6}}{2}, -\sqrt{3}, -\frac{3\sqrt{2}}{2} \right]$$

```
> f &@ p1;
```

$$0$$

```
> f &@ p2;
```

$$0$$

```
> f &@ p3;
```

$$0$$

```
> f &@ p4;
```

$$0$$

```
> f &@ p5;
```

$$\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p6;
```

$$-\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p7;
```

$$\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p8;
```

$$-\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p9;
```

$$-\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p10;
```

$$\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p11;
```

$$-\frac{81\sqrt{6}\sqrt{2}}{8}$$

```
> f &@ p12;
```

$$\frac{81\sqrt{6}\sqrt{2}}{8}$$

So the maximum occurs at

```
> p5;p7;p10;p12;
```

$$\left[\frac{\sqrt{6}}{2}, \sqrt{3}, \frac{3\sqrt{2}}{2} \right]$$

$$\left[\frac{\sqrt{6}}{2}, -\sqrt{3}, \frac{3\sqrt{2}}{2}\right]$$

$$\left[-\frac{\sqrt{6}}{2}, \sqrt{3}, -\frac{3\sqrt{2}}{2}\right]$$

$$\left[-\frac{\sqrt{6}}{2}, -\sqrt{3}, -\frac{3\sqrt{2}}{2}\right]$$

where the function value is

```
> simplify(f &@ p5);
```

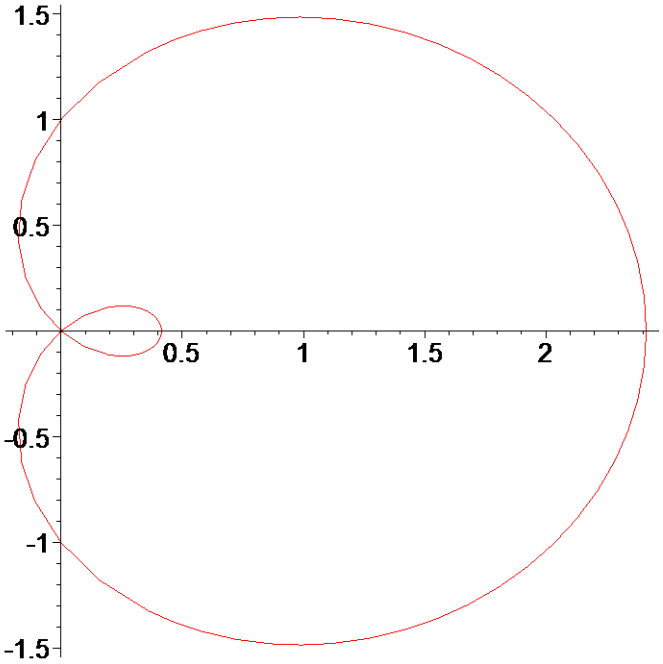
$$\frac{81\sqrt{3}}{4}$$

#2

```
> r1:=sqrt(2)*cos(theta)-1;
```

$$r1 := \sqrt{2}\cos(\theta) - 1$$

```
> polarplot(r1,theta=0..2*Pi);
```




```
> theta0:=solve(r1=0,theta);
```

$$\theta 0 := \frac{\pi}{4}$$

```
> x1:=r*cos(theta);
```

$$x1 := r\cos(\theta)$$

```
> y1:=r*sin(theta);
```

$$y1 := r\sin(\theta)$$

```
> rho:=2+y1;
```

$$\rho := 2 + r\sin(\theta)$$

```
> M:=Muint(rho*r, r=0..r1, theta=-theta0..theta0); M:=value(%);
```

$$M := \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)-1} (2 + r\sin(\theta))\, r\, dr\, d\theta$$

$$M := \pi - 3$$

```
> xmom:=Muint(x1*rho*r, r=0..r1, theta=-theta0..theta0);
  xmom:=value(%);
```

$$xmom := \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)-1} r^2 \cos(\theta)\,(2 + r\sin(\theta))\, dr\, d\theta$$

$$xmom := -\frac{7\sqrt{2}}{3} + \frac{3\sqrt{2}\,\pi}{4}$$

```
> xbar:=xmom/M; xbar:=evalf(%);
```

$$xbar := \frac{-\frac{7\sqrt{2}}{3} + \frac{3\sqrt{2}\,\pi}{4}}{\pi - 3}$$

$$xbar := 0.2283350095$$

```
> ymom:=Muint(y1*rho*r, r=0..r1, theta=-theta0..theta0);
  ymom:=value(%);
```

$$ymom := \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)-1} r^2 \sin(\theta)\,(2 + r\sin(\theta))\, dr\, d\theta$$

$$ymom := \frac{9\,\pi}{32} - \frac{53}{60}$$

```
> ybar:=ymom/M; ybar:=evalf(%);
```

$$ybar := \frac{\frac{9\,\pi}{32} - \frac{53}{60}}{\pi - 3}$$

$$ybar := 0.001692182421$$

```
> cm:=[xbar,ybar];
```

$$cm := [0.2283350095, 0.001692182421]$$

```
> cmp:=r2p(cm);
```

$$cmp := [0.2283412798, 0.007410827828]$$

```
> r2d(cmp[2]);
```

$$0.4246091571$$

```
>
```