Name________________________ Sec__________

1-15	/60	17	/15
16	/15	18	/15
Total		/105	

MATH 251/253 Exam 1 Spring 2008

Sections 508,200,501,502 Version B P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. The triangle with vertices $A = (4,1,5)$, $B = (2,3,4)$ and $C = (3,5,2)$ is

a. equilateral

b. isosceles but not right

c. right but not isosceles

d. isosceles and right

e. scalene

2. Find the area of the triangle with vertices $A = (4,1,5)$, $B = (2,3,4)$ and $C = (3,5,2)$.

a. $\frac{65}{2}$

b. 65

c. 130

d. $\frac{1}{2}\sqrt{65}$

e. $\sqrt{65}$

3. Find an equation of the plane containing the points $A = (4, 1, 5)$, $B = (2, 3, 4)$ and $C = (3, 5, 2)$.

a. $2x + 5y + 6z = 43$

b. $2x - 5y + 6z = 33$

c. $2x - 5y - 6z = -27$

d. $2x + 5y - 6z = -17$

e. $2x - 5y + 6z = 13$

4. Find the point where the line $\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-4}{4}$ intersects the plane $x + y - z = 2$.
Then $x + y + z =$

a. 2

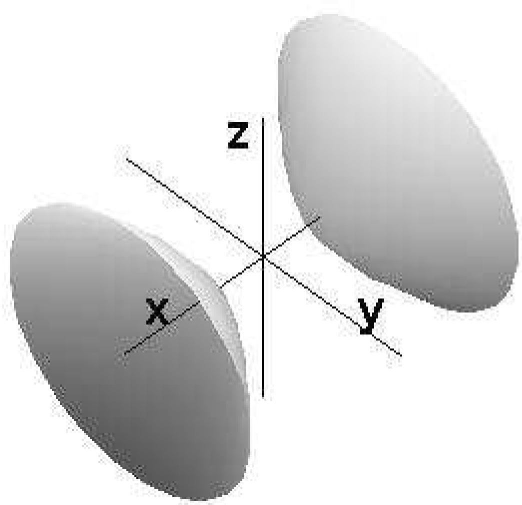
b. 14

c. 18

d. 22

e. 28

5. Which of the following is the equation of the surface?



a. $-x^2 + y^2 + z^2 = -1$

b. $-x^2 + y^2 + z^2 = 0$

c. $-x^2 + y^2 + z^2 = 1$

d. $x + y^2 + z^2 = 0$

e. $x - y^2 - z^2 = 0$

6. Find the line tangent to the curve $\vec{r}(t) = (3t, 3t^2, 2t^3)$ at the point $(3,3,2)$.

a. $(x,y,z) = (3+3t, 3+3t^2, 2+2t^3)$

b. $(x,y,z) = (3+3t, 3-6t^2, 2+6t^3)$

c. $(x,y,z) = (3+3t, 3+6t^2, 2+6t^3)$

d. $(x,y,z) = (3+3t, 3+6t, 2+6t)$

e. $(x,y,z) = (3+3t, 3-6t, 2+6t)$

7. Find the arc length of the curve $\vec{r}(t) = (3t, 3t^2, 2t^3)$ between $(0,0,0)$ and $(3,3,2)$.

a. 1

b. 2

c. 3

d. 4

e. 5

8. Find the tangential acceleration of the curve $\vec{r}(t) = (3t, 3t^2, 2t^3)$.

a. $3t - 2t^3$

b. $3t + 2t^3$

c. $12t$

d. $36t$

e. $6t$

9. A jet fighter flies along the parabola $z = x^2$ in the xz-plane toward increasing values of x. Then, . . .

HINT: There are no computations.

a. $\hat{N} = (0, 0, 1)$ at all times.

b. $\hat{N} = (0, 0, -1)$ at all times.

c. $\hat{N} = (1, 0, 1)$ at all times.

d. $\hat{B} = (0, 1, 0)$ at all times.

e. $\hat{B} = (0, -1, 0)$ at all times.

10. If $f(x, y) = x^2 e^{xy}$, which of the following is FALSE?

a. $f_x(2, 1) = 8e^2$

b. $f_y(2, 1) = 8e^2$

c. $f_{xx}(2, 1) = 14e^2$

d. $f_{yy}(2, 1) = 4e^2$

e. $f_{xy}(2, 1) = 20e^2$

11. Find the plane tangent to the graph of the function $f(x,y) = x^3y^2$ at $(x,y) = (2,1)$.
The z-intercept is

a. -40

b. $+32$

c. -32

d. -8

e. $+8$

12. Find the unit vector direction in which the function $f(x,y) = x^3y^2$ increases most rapidly at the point $(x,y) = (2,1)$.

a. $\left(\frac{3}{5},\frac{4}{5}\right)$

b. $\left(-\frac{3}{5},-\frac{4}{5}\right)$

c. $\left(\frac{4}{5},\frac{3}{5}\right)$

d. $\left(-\frac{4}{5},-\frac{3}{5}\right)$

e. $\left(\frac{4}{5},-\frac{3}{5}\right)$

13. Find an equation of the plane tangent to the surface $x^2z + yz^3 = 11$ at the point $(x,y,z) = (3,2,1)$.

a. $6x - y + 15z = 31$

b. $6x + y + 15z = 35$

c. $3x - 2y + z = 6$

d. $18x - 2y + 15z = 65$

e. $3x + 2y + z = 14$

14. An arch has the shape of the semi-circle $x^2 + y^2 = 16$ for $y \geq 0$ and has linear mass density given by $\rho = 8 - y$ so it is less dense at the top.

Find the total mass of the arch.

NOTE: The arch may be parametrized by $\vec{r}(t) = (4\cos t, 4\sin t)$.

a. $32\pi - 16$

b. $32\pi - 32$

c. $32\pi - 64$

d. 32π

e. $24\pi - 16$

15. Find the center of mass of the arch of problem 14.

a. $\left(0, \dfrac{4-\pi}{\pi-2}\right)$

b. $\left(0, \dfrac{4-\pi}{\pi-1}\right)$

c. $\left(0, \dfrac{4-\pi}{2\pi-1}\right)$

d. $\left(0, \dfrac{8-\pi}{2\pi-1}\right)$

e. $\left(0, \dfrac{8-\pi}{\pi-1}\right)$

Work Out: (15 points each. Part credit possible. Show all work.)

16. An object moves around **2 loops** of the helix $\vec{r}(t) = (4\cos t, 4\sin t, 3t)$

from $(4,0,0)$ to $(4,0,12\pi)$ under the action of a force $\vec{F} = (-y, x, z)$.

Find the work done by the force.

17. A cardboard box has length $L = 50$ cm, width $W = 40$ cm and height $H = 30$ cm.

The cardboard is 0.2 cm thick on each side and 0.4 cm thick on the top and bottom.

Use differentials to estimate the volume of the cardboard used to make the box.

18. In a particular ideal gas the pressure, P, the temperature, T, and density, ρ, are related by $P = 10\rho T$.

Currently, the temperature is $T = 300°$K and decreasing at $2°$K/hr while the density is $\rho = 2 \times 10^{-4}$ gm/cm^3 and increasing at 4×10^{-6} gm/cm^3/hr.

Find the current pressure. Is it increasing or decreasing and at what rate?
