1. Find a parametric equation of the line tangent to the curve \( \vec{r}(\theta) = (2 \sin \theta, 2 \cos \theta, \theta) \) at the point \( (0, -2, \pi) \).

   a. \( X(t) = (-2, -2t, 1 + \pi t) \)
   
   b. \( X(t) = (0, -2t - 2, \pi + t) \)
   
   c. \( X(t) = (-2t, -2, \pi + t) \)
   
   d. \( X(t) = (-2t - 2, 0, \pi + t) \)
   
   e. \( X(t) = (0, -2t - 2, 1 + \pi t) \)

2. The density of the fog is given by \( \rho = 30 - x^2 - y^2 - z \). If an airplane is at the position \( (x, y, z) = (\sqrt{2}, 2, 4) \), in what unit vector direction should the airplane initially travel to get out of the fog as quickly as possible?

   a. \( \left( \frac{-2 \sqrt{2}}{5}, \frac{-4}{5}, \frac{-1}{5} \right) \)
   
   b. \( \left( \frac{2 \sqrt{2}}{5}, \frac{4}{5}, \frac{1}{5} \right) \)
   
   c. \( \left( \frac{-\sqrt{5}}{\sqrt{10}}, \frac{-2}{\sqrt{10}}, \frac{-2}{\sqrt{10}} \right) \)
   
   d. \( \left( \frac{\sqrt{5}}{\sqrt{10}}, \frac{2}{\sqrt{10}}, \frac{2}{\sqrt{10}} \right) \)
   
   e. \( \left( \frac{-\sqrt{5}}{\sqrt{10}}, \frac{2}{\sqrt{10}}, \frac{2}{\sqrt{10}} \right) \)
3. Find an equation of the plane tangent to the graph of the function \( z = x^2y + xy^2 \) at the point \((2, 1)\).
   a. \( z = 5x + 8y + 6 \)
   b. \( z = -5x - 8y + 6 \)
   c. \( z = -5x - 8y + 24 \)
   d. \( z = 5x + 8y - 12 \)
   e. \( z = 5x + 8y - 6 \)

4. Find an equation of the plane tangent to the level surface \( x^2y^2 + x^2z^2 + y^2z^2 = 49 \) at the point \((1, 2, 3)\).
   a. \( 13x + 20y + 15z = 98 \)
   b. \( 13x + 10y + 5z = 48 \)
   c. \( 13x - 10y + 5z = 8 \)
   d. \( 13x - 20y + 15z = 18 \)
   e. \( 39x + 20y + 5z = 94 \)

5. Find the equation of the plane tangent to the parametric surface \( \vec{R}(r, \theta) = (r\cos \theta, r\sin \theta, \theta) \) at the point where \((r, \theta) = (2, \pi)\).
   a. \( -x = -2y + z \)
   b. \( -x + 2y - z = -2 + \pi \)
   c. \( -x - 2y - z = -2 + \pi \)
   d. \( -y + 2z = 2\pi \)
   e. \( y + 2z = 2\pi \)
6. A satellite is travelling from East to West directly above the equator. In what direction does the binormal \( \mathbf{B} \) point?

a. North
b. South
c. Up
d. Down
e. West

7. Compute \( \int_{P}^{Q} 2x \, dx + 2y \, dy + 2z \, dz \) along the straight line from \( P = (1, -2, 2) \) to \( Q = (3, -4, 12) \).

HINT: Use the Fundamental Theorem of Calculus for Curves.

a. \(-10\)
b. \(\sqrt{10}\)
c. 10
d. 108
e. 160

8. Compute \( \oint 2x \, dx + 2xy \, dy \) counterclockwise around the boundary of the rectangle \( 2 \leq x \leq 4, 1 \leq y \leq 4 \).

HINT: Use Green’s Theorem.

a. 6
b. 18
c. 30
d. 36
e. 72
9. Compute \( \iiint_{V} \vec{F} \cdot d\vec{S} \) over the complete boundary of the solid above the paraboloid \( z = x^2 + y^2 \) and below the plane \( z = 4 \) with outward normal, for the vector field \( \vec{F} = (xy^2, yx^2, z^2) \).
HINT: Use Gauss’ Theorem.

\[
\begin{align*}
a. & \quad -\frac{128}{3} \pi \\
b. & \quad 40 \pi \\
c. & \quad 72 \pi \\
d. & \quad \frac{160}{3} \pi \\
e. & \quad \frac{896}{15} \pi \\
\end{align*}
\]

10. Find the area of one petal of the 4 leaf rose \( r = \sin(4\theta) \).
The petal in the first quadrant is shown.

\[
\begin{align*}
a. & \quad \frac{\pi}{16} \\
b. & \quad \frac{\pi}{8} \\
c. & \quad \frac{\pi}{4} \\
d. & \quad \frac{\pi}{2} \\
e. & \quad \pi \\
\end{align*}
\]
11. (15 points) Find the point in the first octant on the graph of $4x^4y^2z = 1$ which is closest to the origin. HINTS: What is the square of the distance from a point to the origin? Lagrange multipliers are easier.
12. (25 points) Verify Stokes' Theorem \[ \oint_C \nabla \times \vec{F} \cdot d\vec{S} = \int_C \nabla \cdot \vec{F} \cdot ds \]

for the cone \( C \) given by \( z^2 = x^2 + y^2 \) for \( z \leq 2 \)

oriented down and out, and the vector field \( \vec{F} = (yz^2, -xz^2, z^3) \).

Be sure to check and explain the orientations.

Use the following steps:

a. Note: The cone may be parametrized as \( \vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, r) \)

Compute the surface integral by successively finding:

\( \vec{e}_r, \vec{e}_\theta, \vec{N}, \nabla \times \vec{F}, \nabla \times \vec{F}(\vec{R}(r, \theta)), \oint_C \nabla \times \vec{F} \cdot d\vec{S} \)
Recall $\vec{F} = (yz^2, -xz^2, z^3)$.

b. Compute the line integral by parametrizing the boundary curve and successively finding:

$r(\theta), \quad \vec{v}, \quad \vec{F}(r(\theta)), \quad \oint_{\partial C} \vec{F} \cdot d\vec{s}$

13. (15 points) Find the mass and center of mass of the $\frac{1}{8}$ of the sphere $x^2 + y^2 + z^2 \leq 4$ in the first octant if the density is $\delta = x^2 + y^2 + z^2$. 