Name______________________ Sec__________

1-9	/54	11c	/10
10	/10	11d	/10
11a	/10	11e	/ 6
11b	/10	Total	/104

MATH 253 Exam 2 Spring 2009

Sections 200,501,502 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Find the volume under $z = xy^2$ above the rectangle $1 \le x \le 2$ and $0 \le y \le 2$.

a. 2

b. 4

c. 6

d. 12

e. $\frac{16}{3}$

2. (Non-Honors Only) Compute $\int_0^2 \int_0^y \int_x^y x \, dz \, dx \, dy$.

a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. $\frac{3}{4}$

d. $\frac{4}{5}$

e. $\frac{5}{6}$

3. Compute $\iiint_R z\,dV$ over the region R in the first octant bounded by $y = 9 - x^2$, $z = 2$ and the coordinate planes.

a. 36

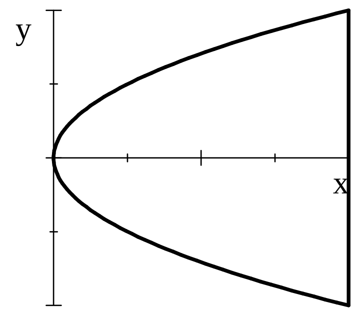
b. 54

c. 72

d. 96

e. 108

4. Find the mass of the plate bounded by the curves $x = y^2$ and $x = 4$, if the surface mass density is $\rho = x$.



a. $\frac{32}{3}$

b. $\frac{64}{3}$

c. $\frac{128}{3}$

d. $\frac{64}{5}$

e. $\frac{128}{5}$

5. Find the center of mass of the plate bounded by the curves $x = y^2$ and $x = 4$, if the surface mass density is $\rho = x$.

a. $(\bar{x},\bar{y}) = \left(\frac{12}{7},0\right)$

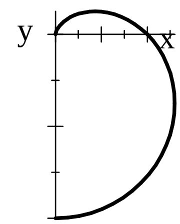
b. $(\bar{x},\bar{y}) = (2,0)$

c. $(\bar{x},\bar{y}) = \left(\frac{20}{7},0\right)$

d. $(\bar{x},\bar{y}) = \left(\frac{24}{7},0\right)$

e. $(\bar{x},\bar{y}) = \left(\frac{512}{7},0\right)$

6. A styrofoam board is cut in the shape of the right half of the cardioid $r = 1 - \sin\theta$.
A static electricity charge is put on the board whose surface charge density is given by $\rho_e = x$.
Find the total charge on the board $Q = \iint \rho_e \, dA$.



a. 0

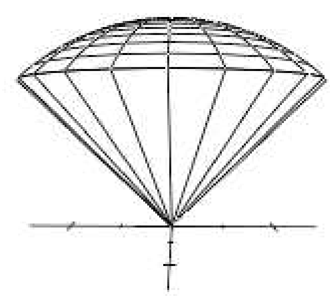
b. $\frac{2}{3}$

c. $\frac{4}{3}$

d. $\frac{8\pi}{3}$

e. $\frac{16\pi}{3}$

7. Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ below the hemisphere $x^2 + y^2 + z^2 = 4$.



a. $\frac{8\pi}{3}$

b. $\frac{8\pi}{3}\sqrt{2}$

c. $\frac{16\pi}{3}$

d. $\frac{8\pi}{3}\left(2 - \sqrt{2}\right)$

e. $\frac{8\pi}{3}\left(2 + \sqrt{2}\right)$

8. Compute $\int_0^2 \int_{y^2}^4 y e^{x^2} \, dx \, dy$. HINT: Interchange the order of integration.

a. $\frac{1}{4} e^{16}$

b. $\frac{1}{4}(e^{16} - 1)$

c. $\frac{1}{2}(1 - e^{16})$

d. $\frac{1}{4}(e^{4} - 1)$

e. $\frac{1}{2}(1 - e^{4})$

9. Compute $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z \cos\left[(x^2 + y^2 + z^2)^2 \right] dz \, dy \, dx$. HINT: Convert to spherical coordinates.

a. $\frac{\pi}{8} \sin(4)$

b. $\frac{\pi}{16} \sin(4)$

c. $\frac{\pi}{4} \sin(16)$

d. $\frac{\pi}{8} \sin(16)$

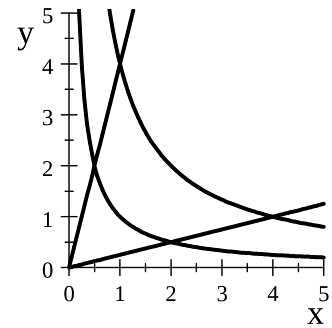
e. $\frac{\pi}{16} \sin(16)$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (10 points) Compute $\iint y\,dx\,dy$ over the diamond shaped region bounded by the curves

$$y = 4x \qquad y = \frac{x}{4} \qquad y = \frac{1}{x} \qquad y = \frac{4}{x}$$

HINT: Let $u^2 = xy$ and $v^2 = \dfrac{y}{x}$. Solve for x and y.



11. Consider the surface, S, given parametrically by

$$\vec{R}(p,q) = \left(\frac{1}{2}p^2, q^2, pq\right) \quad \text{for} \quad 0 \le p \le 3 \quad \text{and} \quad 0 \le q \le 2.$$

a. (10 points) Find $\vec{e}_p$, $\vec{e}_q$, $\vec{N}$, and $\left|\vec{N}\right|$. Simplify $\left|\vec{N}\right|$ by looking for a perfect square.

b. (10 points) Compute the surface area of the surface, S.
HINT: $A = \iint 1\, dS$

Recall $\vec{R}(p,q) = \left(\frac{1}{2}p^2, q^2, pq\right)$ for $0 \le p \le 3$ and $0 \le q \le 2$.

c. (10 points) Compute the mass of the surface, S, if the surface mass density is $\rho(x,y,z) = z$.
HINT: $M = \iint \rho\, dS$

d. (10 points) Compute the flux through the surface, S, of the vector field $\vec{F} = (2x, 2y, z)$ if the surface is oriented down and out.
HINT: $Flux = \iint \vec{F} \cdot d\vec{S}$

e. (6 points HONORS ONLY) Find the equation of the plane tangent to the surface, S, at the point where $(p,q) = (2,1)$.