Name________________________ Sec_________

1-10	/50	14	/20
12	/10	15	/ 5
13	/20	Total	/105

MATH 253 Final Exam Spring 2009

Sections 200,501,502 Solutions P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_{(e^{-1},1,e^{1})}^{(2e^{-2},4,8e^{2})} \vec{F} \cdot d\vec{s}$ where $\vec{F} = (yz,xz,xy)$ along the curve $\vec{r}(t) = (te^{-t},t^2,t^3e^t)$.

HINT: Find a scalar potential for $\vec{F}$. Use the Fundamental Theorem of Calculus for Curves.

a. $2e^{-3} - 4 + 8e^3$

b. $2e^{-3} + 4 + 8e^3$

c. $2e^{-2} - 6e^{-1} + 12e - 8e^2$

d. 63 Correct Choice

e. 64

$\vec{F} = (yz,xz,xy) = \vec{\nabla}f$ for $f = xyz$. So

$$\int_{(e^{-1},1,e^{1})}^{(2e^{-2},4,8e^{2})} \vec{F} \cdot d\vec{s} = \int_{(e^{-1},1,e^{1})}^{(2e^{-2},4,8e^{2})} \vec{\nabla}f \cdot d\vec{s} = f(2e^{-2},4,8e^2) - f(e^{-1},1,e^1) = 64 - 1 = 63$$

2. Compute $\oint (\sin(x^3) - 2x^2y)\,dx + (2xy^2 + \cos(y^3))\,dy$ counterclockwise around the circle $x^2 + y^2 = 9$.

HINT: Use Green's Theorem.

a. 81π Correct Choice

b. 36π

c. 18π

d. 9π

e. 3π

By Green's Theorem,

$$\oint (\sin(x^3) - 2x^2y)\,dx + (2xy^2 + \cos(y^3))\,dy = \iint \left(\frac{\partial}{\partial x}(2xy^2 + \cos(y^3)) - \frac{\partial}{\partial y}(\sin(x^3) - 2x^2y) \right) dx\,dy$$

$$= \iint (2y^2 + 2x^2)\,dx\,dy = \int_0^{2\pi}\int_0^3 2r^2 r\,dr\,d\theta = 2\pi\left[\frac{r^4}{2}\right]_{r=0}^{3} = 81\pi$$

3. Find the equation of the line perpendicular to the hyperboloid $xyz = 6$ at the point $(3,2,1)$.

a. $2x + 3y + 6z = 18$

b. $3x + 2y + z = 18$

c. $(x,y,z) = (3+2t, 2-3t, 1+6t)$

d. $(x,y,z) = (2+3t, 3+2t, 6+t)$

e. $(x,y,z) = (3+2t, 2+3t, 1+6t)$ Correct Choice

$P = (3,2,1) \qquad F = xyz \qquad \vec{\nabla}F = (yz, xz, xy) \qquad \vec{N} = \vec{\nabla}F\Big|_{(3,2,1)} = (2,3,6)$

$X = P + t\vec{N} = (3,2,1) + t(2,3,6) = (3+2t, 2+3t, 1+6t)$

4. Find the mass of the "twisted cubic" curve
$\vec{r}(t) = \left(\frac{2}{3}t^3, t^2, t\right)$ for $0 \le t \le 1$ if the density is $\rho = 3xz + 3y^2$.

a. $M = \dfrac{5}{3}$

b. $M = \dfrac{17}{7}$ Correct Choice

c. $M = \dfrac{86}{21}$

d. $M = \dfrac{111}{35}$

e. $M = 1$

$\vec{v} = (2t^2, 2t, 1) \qquad |\vec{v}| = \sqrt{4t^4 + 4t^2 + 1} = 2t^2 + 1 \qquad \rho = 3\frac{2}{3}t^3 t + 3(t^2)^2 = 5t^4$

$M = \int \rho\, ds = \int_0^1 \rho |\vec{v}|\, dt = \int_0^1 5t^4(2t^2+1)\, dt = 5\int_0^1 (2t^6 + t^4)\, dt = 5\left[2\frac{t^7}{7} + \frac{t^5}{5}\right]_0^1 = 5\left(\frac{2}{7} + \frac{1}{5}\right) = \frac{17}{7}$

5. Find the z-component of the center of mass of the "twisted cubic" curve
$\vec{r}(t) = \left(\frac{2}{3}t^3, t^2, t\right)$ for $0 \le t \le 1$ if the density is $\rho = 3xz + 3y^2$.

a. $\bar{z} = \dfrac{1}{2}$

b. $\bar{z} = \dfrac{204}{175}$

c. $\bar{z} = \dfrac{175}{204}$ Correct Choice

d. $\bar{z} = \dfrac{25}{12}$

e. $\bar{z} = \dfrac{12}{25}$

$z\text{-mom} = M_{xy} = \int z\rho\, ds = \int_0^1 z\rho|\vec{v}|\, dt = \int_0^1 t\,5t^4(2t^2+1)\, dt = 5\int_0^1 (2t^7 + t^5)\, dt$

$= 5\left[\frac{t^8}{4} + \frac{t^6}{6}\right]_0^1 = 5\left(\frac{1}{4} + \frac{1}{6}\right) = \frac{25}{12} \qquad \bar{z} = \frac{z\text{-mom}}{M} = \frac{M_{xy}}{M} = \frac{25}{12}\frac{7}{17} = \frac{175}{204}$

6. The point $(1,2)$ is a critical point of the function $f(x,y) = 16x^4 + y^4 - 32xy$.
Classify the point $(1,2)$ using the Second Derivative Test.

a. Local Mininum Correct Choice

b. Local Maximum

c. Saddle Point

d. Inflection Point

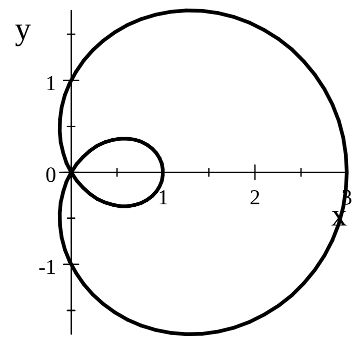
e. Test Fails

$f_x = 64x^3 - 32y \quad f_x(1,2) = 0 \quad f_y = 4y^3 - 32x \quad f_y(1,2) = 0$

$f_{xx} = 192x^2 \quad f_{xx}(1,2) = 192 > 0 \quad f_{yy} = 12y^2 \quad f_{yy}(1,2) = 48 \quad f_{xy} = -32 \quad f_{xy}(1,2) = -32$

$D = f_{xx}f_{yy} - f_{xy}{}^2 \quad D(1,2) = 192 \bullet 48 - 32^2 = 8192 > 0$ Local Minimum

7. Find the area inside the small loop of the limacon $r = -1 + 2\cos\theta$.



a. $2\sqrt{3} - \frac{2}{3}\pi$

b. $\sqrt{3} - \frac{1}{3}\pi$

c. $\frac{1}{2}\pi + \frac{1}{2}\sqrt{3} - 2$

d. $\pi - \frac{3}{2}\sqrt{3}$ Correct Choice

e. $\frac{\pi}{2} - \frac{3}{4}\sqrt{3}$

Find the angles where $r \to 0$: $\quad -1 + 2\cos\theta = 0 \quad \Rightarrow \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \pm 60° = \pm\frac{\pi}{3}$

$$A = \iint 1\,dA = \int_{-\pi/3}^{\pi/3}\int_0^{-1+2\cos\theta} r\,dr\,d\theta = \int_{-\pi/3}^{\pi/3}\left[\frac{r^2}{2}\right]_0^{-1+2\cos\theta} d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}(-1+2\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(1 - 4\cos\theta + 4\cos^2\theta)\,d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}(1 - 4\cos\theta + 2(1+\cos 2\theta))\,d\theta = \frac{1}{2}\Big[3\theta - 4\sin\theta + \sin 2\theta\Big]_{-\pi/3}^{\pi/3}$$

$$= \left(3\frac{\pi}{3} - 4\sin\frac{\pi}{3} + \sin 2\frac{\pi}{3}\right) = \pi - 2\sqrt{3} + \frac{\sqrt{3}}{2} = \pi - \frac{3}{2}\sqrt{3}$$

8. (Non-Honors Only) Find the mass of the hollow solid between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ if the density is $\delta = \dfrac{1}{x^2 + y^2 + z^2}$.

a. 2π

b. 4π Correct Choice

c. 10π

d. $\frac{\pi^2}{3}$

e. $\frac{2\pi}{3}$

$$M = \iiint \delta\,dV = \int_0^{2\pi}\int_0^{\pi}\int_2^3 \frac{1}{\rho^2}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 2\pi(3-2)\Big[-\cos\varphi\Big]_0^{\pi} = 2\pi(--1--1) = 4\pi$$

9. Duke Skywater is flying the Millenium Eagle through the galaxy. Currently, he is at the point $P = (1,2,3)$ and has velocity $\vec{v} = (4,5,6)$. He discovers that he is passing through a dangerous polaron field which has density $\rho = x^3y^2z$. What is the rate of change of the polaron density as seen by Duke at this instant?

a. 228 Correct Choice

b. 144

c. 72

d. 32

e. 12

$\vec{\nabla}\rho = (3x^2y^2z, 2x^3yz, x^3y^2)$ $\quad \vec{\nabla}\rho\big|_P = (36,12,4)$ $\quad \dfrac{d\rho}{dt} = \vec{v}\cdot\vec{\nabla}\rho\big|_P = 4\cdot 36 + 5\cdot 12 + 6\cdot 4 = 228$

10. Duke Skywater is flying the Millenium Eagle through the galaxy. Currently, he is at the point $P = (1,2,3)$ and has velocity $\vec{v} = (4,5,6)$. He discovers that he is passing through a dangerous polaron field which has density $\rho = x^3y^2z$. In what unit vector direction should Duke travel to decrease the polaron density as fast as possible?

a. $\dfrac{1}{\sqrt{91}}(9,3,1)$

b. $(9,-3,1)$

c. $\dfrac{1}{\sqrt{91}}(9,-3,1)$

d. $(-9,-3,-1)$

e. $\dfrac{1}{\sqrt{91}}(-9,-3,-1)$ Correct Choice

A function decreases fastest in the direction $-\vec{\nabla}\rho\big|_P = (-36,-12,-4)$

11. (Honors Only) Han Duet is flying the Millenium Eagle through the galaxy along the path $\vec{r}(t) = (t,t^2,t^3)$ with velocity $\vec{v} = \dfrac{d\vec{r}}{dt}$. At time $t = 2$ hours he releases a garbage pod which travels along his tangent line with constant velocity equal to $\vec{v}(2)$, the velocity of the Eagle when the pod was released. Where is the pod at $t = 5$ hours, which is 3 hours after its release?

a. $(3,9,27)$

b. $(4,12,32)$

c. $(5,16,44)$ Correct Choice

d. $(6,20,56)$

e. $(7,24,68)$

$\vec{r}(t) = (t,t^2,t^3)$ $\quad \vec{v} = (1,2t,3t^2)$ $\quad P = \vec{r}(2) = (2,4,8)$ $\quad \vec{v}(2) = (1,4,12)$

$X(T) = P + Tv(2) = (2,4,8) + T(1,4,12) = (2+T, 4+4T, 8+12T)$

$t = 2$ when $T = 0$ $\qquad t = 5$ when $T = 3$

$X(3) = (2+3, 4+4\cdot 3, 8+12\cdot 3) = (5,16,44)$

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (10 points) Find the point (x,y,z) in the first octant on the surface $z = \frac{27}{5x} + \frac{64}{5y}$ which is closest to the origin.

Minimize the square of the distance to the origin $f = x^2 + y^2 + z^2$

subject to the constraint that the point lies on the surface $z = \frac{27}{5x} + \frac{64}{5y}$.

Method 1: Eliminate a constraint: Minimize $f = x^2 + y^2 + \left(\frac{27}{5x} + \frac{64}{5y}\right)^2$

$f_x = 2x + 2\left(\frac{27}{5x} + \frac{64}{5y}\right)\left(\frac{-27}{5x^2}\right) = 0 \qquad f_y = 2y + 2\left(\frac{27}{5x} + \frac{64}{5y}\right)\left(\frac{-64}{5y^2}\right) = 0$

Multiply the first equation by $\frac{5x^2}{54}$ and the second equation by $\frac{5y^2}{128}$:

(1) $\frac{5x^3}{27} = \left(\frac{27}{5x} + \frac{64}{5y}\right)$ (2) $\frac{5y^3}{64} = \left(\frac{27}{5x} + \frac{64}{5y}\right)$

Equate these to obtain $\frac{x}{3} = \frac{y}{4}$ and plug back into (1) to obtain:

$\frac{5x^3}{27} = \left(\frac{27}{5x} + \frac{16 \cdot 3}{5x}\right) = \frac{75}{5x}$

Cross multiply: $5^2x^4 = 75 \cdot 27 = 3^4 \cdot 5^2$ So $x = 3$ and $y = 4$

and $z = \frac{27}{5x} + \frac{64}{5y} = \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$.

Method 2: Lagrange Multipliers: The constraint is $g = z - \frac{27}{5x} - \frac{64}{5y} = 0$.

$\vec{\nabla} f = (2x, 2y, 2z) \qquad \vec{\nabla} g = \left(\frac{27}{5x^2}, \frac{64}{5y^2}, 1\right) \qquad \vec{\nabla} f = \lambda \vec{\nabla} g$

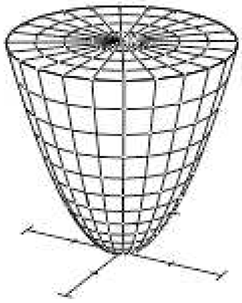
$2x = \lambda \frac{27}{5x^2} \qquad 2y = \lambda \frac{64}{5y^2} \qquad 2z = \lambda$ Eliminate λ: $2x = 2z\frac{27}{5x^2} \qquad 2y = 2z\frac{64}{5y^2}$

Solve for z: $z = \frac{5x^3}{27} = \frac{5y^3}{64}$ So $\frac{x}{3} = \frac{y}{4}$ or $y = \frac{4x}{3}$

Plug into the constraint: $z = \frac{27}{5x} + \frac{64}{5y} \quad \Rightarrow \quad \frac{5x^3}{27} = \frac{27}{5x} + \frac{16 \cdot 3}{5x} = \frac{75}{5x} = \frac{15}{x}$

Cross multiply: $x^4 = 81 \qquad x = 3 \qquad y = 4 \qquad z = \frac{5x^3}{27} = 5$

13. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xz^2, yz^2, -z^3)$ and the solid V
above the paraboloid P given by $z = x^2 + y^2$
or parametrized by $R(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$,
below the disk D given by $x^2 + y^2 \le 4$ and $z = 4$.
Be sure to check and explain the orientations.
Use the following steps:

a. Compute the volume integral by successively finding:

$$\vec{\nabla} \cdot \vec{F}(x,y,z), \quad \vec{\nabla} \cdot \vec{F}(r,\theta,z), \quad dV, \quad \iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$$

$$\vec{\nabla} \cdot \vec{F} = z^2 + z^2 - 3z^2 = -z^2 \qquad dV = r\,dr\,d\theta\,dz$$

$$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \int_0^{2\pi}\int_0^2\int_{r^2}^4 -z^2\, r\,dz\,dr\,d\theta = -2\pi\int_0^2\left[\frac{z^3}{3}\right]_{z=r^2}^4 r\,dr = -2\pi\int_0^2\left(\frac{64}{3} - \frac{r^6}{3}\right) r\,dr$$

$$= \frac{-2\pi}{3}\left[64\frac{r^2}{2} - \frac{r^8}{8}\right]_{r=0}^2 = \frac{-2\pi}{3}(128 - 32) = -64\pi$$

b. Compute the surface integral over the disk by parametrizing the disk and successively finding:

$$\vec{R}(r,\theta), \quad \vec{e}_r, \quad \vec{e}_\theta, \quad \vec{N}, \quad \vec{F}\left(\vec{R}(r,\theta)\right), \quad \iint\limits_D \vec{F} \cdot d\vec{S}$$

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 4)$$

$$\begin{array}{r|ccc|} & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \vec{e}_r = & (\cos\theta, & \sin\theta, & 0) \\ \vec{e}_\theta = & (-r\sin\theta, & r\cos\theta, & 0) \end{array}$$

$$\vec{N} = \vec{e}_r \times \vec{e}_\theta = \hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}(r\cos^2\theta + r\sin^2\theta) = (0,0,r)$$

We need $\vec{N}$ to point up which it does.

$$\vec{F} = (xz^2, yz^2, -z^3) \qquad \vec{F}\left(\vec{R}(r,\theta)\right) = (16r\cos\theta, 16r\sin\theta, -64)$$

$$\iint\limits_D \vec{F} \cdot d\vec{S} = \iint\limits_D \vec{F} \cdot \vec{N}\, dr\,d\theta = \int_0^{2\pi}\int_0^2 -64r\,dr\,d\theta = -128\pi\left[\frac{r^2}{2}\right]_0^2 = -256\pi$$

Recall: $\vec{F} = (xz^2, yz^2, -z^3)$ and P is parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

c. Compute the surface integral over the paraboloid P by successively finding:

$$\vec{e}_r,\quad \vec{e}_\theta,\quad \vec{N},\quad \vec{F}\left(\vec{R}(r,\theta)\right),\quad \iint_P \vec{F}\cdot d\vec{S}$$

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$$

$$\begin{array}{r|ccc|} & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \vec{e}_r = & (\cos\theta, & \sin\theta, & 2r) \\ \vec{e}_\theta = & (-r\sin\theta, & r\cos\theta, & 0) \end{array}$$

$$\vec{N} = \vec{e}_r \times \vec{e}_\theta = \hat{\imath}(-2r^2\cos\theta) - \hat{\jmath}(2r^2\sin\theta) + \hat{k}(r\cos^2\theta + r\sin^2\theta) = (-2r^2\cos\theta, -2r^2\sin\theta, r)$$

We need $\vec{N}$ to point down (out of the volume). Reverse $\vec{N} = (2r^2\cos\theta, 2r^2\sin\theta, -r)$

$$\vec{F} = (xz^2, yz^2, -z^3) \qquad \vec{F}\left(\vec{R}(r,\theta)\right) = (r^5\cos\theta, r^5\sin\theta, -r^6)$$

$$\vec{F}\cdot\vec{N} = 2r^7\cos^2\theta + 2r^7\sin^2\theta + r^7 = 3r^7$$

$$\iint_P \vec{F}\cdot d\vec{S} = \iint_P \vec{F}\cdot\vec{N}\,dr\,d\theta = \int_0^{2\pi}\int_0^2 3r^7\,dr\,d\theta = 2\pi\left[3\frac{r^8}{8}\right]_0^2 = 192\pi$$

d. Combine $\iint_D \vec{F}\cdot d\vec{S}$ and $\iint_P \vec{F}\cdot d\vec{S}$ to get $\iint_{\partial V} \vec{F}\cdot d\vec{S}$

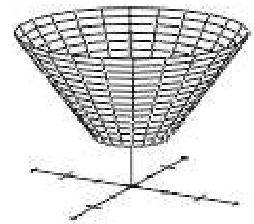
$$\iint_{\partial V} \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot d\vec{S} + \iint_P \vec{F}\cdot d\vec{S} = -256\pi + 192\pi = -64\pi$$

which agrees with part (a).

14. (20 points) Verify Stokes' Theorem $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint\limits_{\partial C} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = (2yz, -2xz, z^2)$ and the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ oriented up and in.

Notice that the boundary of the piece of cone is two circles.
Be sure to check orientations. Use the following steps:



a. The cone may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.
Successively find: $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

$$\begin{array}{rcccc} & & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \vec{e}_r = & (& \cos\theta, & \sin\theta, & 1) \\ \vec{e}_\theta = & (& -r\sin\theta, & r\cos\theta, & 0\ ) \end{array}$$

$$\vec{N} = \vec{e}_r \times \vec{e}_\theta = \hat{\imath}(-r\cos\theta) - \hat{\jmath}(r\sin\theta) + \hat{k}(r\cos^2\theta + r\sin^2\theta) = (-r\cos\theta, -r\sin\theta, r)$$

$\vec{N}$ has the correct orientation.

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2yz & -2xz & z^2 \end{vmatrix} = \hat{\imath}(2x) - \hat{\jmath}(-2y) + \hat{k}(-2z - 2z) = (2x, 2y, -4z)$$

$$\vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right) = (2r\cos\theta, 2r\sin\theta, -4r)$$

$$\vec{\nabla} \times \vec{F} \cdot \vec{N} = -2r^2\cos^2\theta - 2r^2\sin^2\theta - 4r^2 = -6r^2$$

$$\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \iint\limits_C \vec{\nabla} \times \vec{F} \cdot \vec{N}\, dr\, d\theta = \int_0^{2\pi}\int_1^3 -6r^2\, dr\, d\theta = 2\pi[-2r^3]_{r=1}^3 = -104\pi$$

Recall $\vec{F} = (2yz, -2xz, z^2)$

b. Parametrize the upper circle U and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_U \vec{F} \cdot d\vec{s}$

$\vec{r}(\theta) = (3\cos\theta, 3\sin\theta, 3)$

$\vec{v}(\theta) = (-3\sin\theta, 3\cos\theta, 0)$ oriented correctly counterclockwise

$\vec{F}(\vec{r}(\theta)) = (18\sin\theta, -18\cos\theta, 9)$

$$\oint_U \vec{F} \cdot d\vec{s} = \int_0^{2\pi} \vec{F} \cdot \vec{v}\, d\theta = \int_0^{2\pi} -54\sin^2\theta - 54\cos^2\theta\, d\theta = -\int_0^{2\pi} 54\, d\theta = -108\pi$$

c. Parametrize the lower circle L and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_L \vec{F} \cdot d\vec{s}$

$\vec{r}(\theta) = (\cos\theta, \sin\theta, 1)$

$\vec{v}(\theta) = (-\sin\theta, \cos\theta, 0)$ oriented counterclockwise

Rev $\vec{v}(\theta) = (\sin\theta, -\cos\theta, 0)$
$\vec{F}(\vec{r}(\theta)) = (2\sin\theta, -2\cos\theta, 1)$

$$\oint_L \vec{F} \cdot d\vec{s} = \int_0^{2\pi} \vec{F} \cdot \vec{v}\, d\theta = \int_0^{2\pi} 2\sin^2\theta + 2\cos^2\theta\, d\theta = \int_0^{2\pi} 2\, d\theta = 4\pi$$

d. Combine $\oint_U \vec{F} \cdot d\vec{s}$ and $\oint_L \vec{F} \cdot d\vec{s}$ to get $\oint_{\partial C} \vec{F} \cdot d\vec{s}$

$$\oint_{\partial C} \vec{F} \cdot d\vec{s} = \oint_U \vec{F} \cdot d\vec{s}\_ + \oint_L \vec{F} \cdot d\vec{s} = -108\pi + 4\pi = -104\pi$$
which agrees with part (a).

15. (5 points) Mark the Project that you worked on and then answer the questions in 1 or 2 sentences.

___Gauss' Law and Ampere's Law

One of the electric fields produced a charge density of zero: $\rho_c = \frac{1}{4\pi}\vec{\nabla} \cdot \vec{E} = 0$.

Was there a charge and how did you know? Why was Gauss' Theorem not violated?

___Skimpy Donut

For the minimal donut, what was the relation between a and b?

For the maximal donut, what was the value of b?

___Volume Between a Surface and Its Tangent Plane

When minimizing over a square, which tangent point (a,b) minimizes the volume?

___Hypervolume of a Hypersphere

The volume enclosed by a sphere of radius R in $\mathbb{R}^n$ is $V_n = k\pi^p R^q$.

What are the values of p and q when $n = 4$?

What are the values of p and q when $n = 5$?

___Average Temperatures

What was the shape of the probe used to measure the temperature of the water in the pot?

Which Maple command did you use when Maple was unable to compute the integrals?

___Center of Mass of Planet X

In computing the mass of the water, how did you ensure you only integrated where the land level was below sea level?

Which Maple command did you use when Maple was unable to compute the integrals?