Name______________________________ Sec___________

1-11	/66	13	/10
12	/20	14	/10
		Total	/106

MATH 253 Exam 1 Fall 2009

Sections 501,503 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. If $f(x,y) = x^2\cos(y^2)$, which of the following is FALSE?

a. $f_x(x,y) = 2x\cos(y^2)$

b. $f_y(x,y) = -2x^2y\sin(y^2)$

c. $f_{xx}(x,y) = 2\cos(y^2)$

d. $f_{yy}(x,y) = -4x^2y\cos(y^2)$

e. $f_{xy}(x,y) = -4xy\sin(y^2)$

2. The quadratic surface $x^2 - y^2 + z^2 - 4x - 6y - 10z + 16 = 0$ is a

a. hyperboloid of 1 sheet and center $(2,3,5)$

b. hyperboloid of 1 sheet and center $(2,-3,5)$

c. hyperboloid of 2 sheets and center $(2,3,5)$

d. hyperboloid of 2 sheets and center $(2,-3,5)$

e. cone with vertex $(2,3,5)$

3. An airplane is travelling due North at constant speed and a constant altitude as it crosses the equator. In what direction does the $\hat{B}$ vector point?
HINTS: Remember the Earth is curved. Ignore the rotation of the Earth.

a. East

b. West

c. South

d. Up

e. Down

4. A triangle has edge vectors $\overrightarrow{AB} = (2, 1, -2)$ and $\overrightarrow{AC} = (-2, -2, 4)$.
Find the altitude of the triangle if $\overline{AB}$ is the base.

a. $\frac{2\sqrt{5}}{3}$

b. $\frac{\sqrt{5}}{3}$

c. $2\sqrt{5}$

d. $\sqrt{5}$

e. $3\sqrt{5}$

5. A box slides down the helical ramp $\vec{r}(t) = (4\cos t, 4\sin t, 9 - 3t)$ starting at height $z = 9$ and ending at height $z = 0$. How far does the box slide?

a. 3

b. 5

c. 15

d. 25

e. 75

6. A box slides down the helical ramp $\vec{r}(t) = (4\cos t, 4\sin t, 9 - 3t)$ starting at height $z = 9$ and ending at height $z = 0$ under the action of the force $\vec{F} = (-yz, xz, 5z)$.
Find the work done on the box.

a. $\frac{9}{2}$

b. 9

c. $\frac{25}{2}$

d. $\frac{27}{2}$

e. 27

7. The diameter and height of a cylindrical trash can (no lid) are measured as $D = 30$ cm and $h = 40$ cm. The metal is 0.2 cm thick. Use differentials to estimate the volume of metal used to make the can.

a. 165π cm^3

b. 210π cm^3

c. 285π cm^3

d. 330π cm^3

e. 525π cm^3

8. Find the equation of the plane tangent to the surface $z = x^3y^2$ at the point $(2, 1)$. Then the z-intercept is $z =$

a. -40

b. 8

c. -8

d. 32

e. -32

9. Find the equation of the plane tangent to the surface $12xyz - z^3 = 45$ at the point $(1, 2, 3)$. Then the z-intercept is $z =$

a. 135

b. 45

c. $-\sqrt[3]{6}$

d. -45

e. -135

10. Starting from the point $(1,-2)$, find the maximum rate at which the function $f(x,y) = x^2y^3$ increases.

a. 20

b. 25

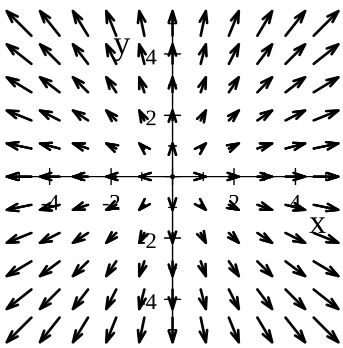
c. 400

d. $(-16, 12)$

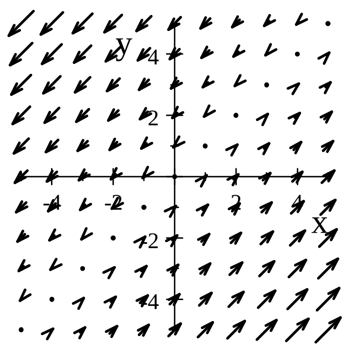
e. $(16,-12)$

11. Which of the following is the plot of the vector field $F(x,y) = (x+y, x-y)$?

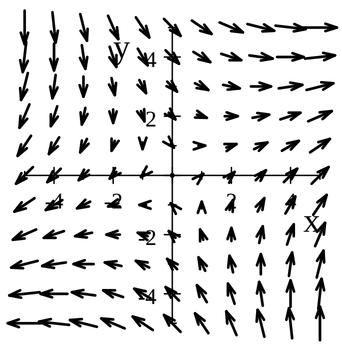
a.



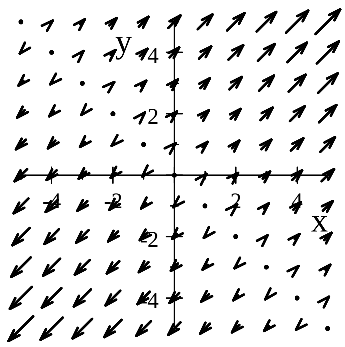
d.



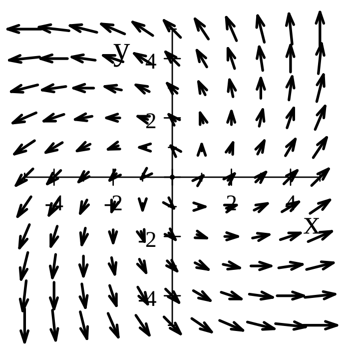
b.



e.



c.



Work Out: (Points indicated. Part credit possible. Show all work.)

12. (20 points) Find the point on the curve $\vec{r}(t) = \left(e^t, \sqrt{2}\,t, e^{-t}\right)$ where the curvature is a local maximum or local minimum. Is it a local maximum or local minimum?

HINTS: First find the curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$. Then find the critical point and apply the first or second derivative test.

13. (10 points) The pressure, P, density, D, and temperature, T, of a certain ideal gas are related by $P = 4DT$. A fly is currently at the point $\vec{r}(t_0) = (3,2,4)$ and has velocity $\vec{v}(t_0) = (2,1,2)$. At the point $(3,2,4)$, the density and temperature and their gradients are

$$D = 50 \qquad \vec{\nabla}D = \left(\frac{\partial D}{\partial x}, \frac{\partial D}{\partial y}, \frac{\partial D}{\partial z}\right) = (0.1, 0.4, 0.2)$$

$$T = 300 \qquad \vec{\nabla}T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) = (2, 3, 1)$$

Find the time rate of change of the pressure, $\frac{dP}{dt}$, as seen by the fly.

14. (10 points) Suppose $p = p(x,y)$, while $x = x(u,v)$ and $y = y(u,v)$.

Further, you know the following information:

$x(2,1) = 3$	$y(2,1) = 4$	$p(2,1) = 13$	$p(3,4) = 14$
$\frac{\partial p}{\partial x}(2,1) = 5$	$\frac{\partial p}{\partial y}(2,1) = 6$	$\frac{\partial p}{\partial x}(3,4) = 7$	$\frac{\partial p}{\partial y}(3,4) = 8$
$\frac{\partial x}{\partial u}(2,1) = 9$	$\frac{\partial x}{\partial v}(2,1) = 10$	$\frac{\partial x}{\partial u}(3,4) = 11$	$\frac{\partial x}{\partial v}(3,4) = 12$
$\frac{\partial y}{\partial u}(2,1) = 18$	$\frac{\partial y}{\partial v}(2,1) = 17$	$\frac{\partial y}{\partial u}(3,4) = 16$	$\frac{\partial y}{\partial v}(3,4) = 15$

a. Write out the chain rule for $\frac{\partial p}{\partial u}$.

b. Then use it and the above information to compute $\frac{\partial p}{\partial u}(2,1)$.