Name______________________________ Sec_______

1-12	/60	15	/15
13	/15	16	/ 5
14	/10	Total	/105

MATH 253 Exam 2 Fall 2009

Section 501,503 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^2 \int_{-x}^{x} 3y^2 \, dy \, dx$.

a. 8

b. 12

c. 16

d. 24

e. 0

2. Compute $\int_0^9 \int_{\sqrt{y}}^{3} \pi \sin(\pi x^3) \, dx \, dy$. HINT: Reverse the order of integration.

a. $\frac{1}{9}$

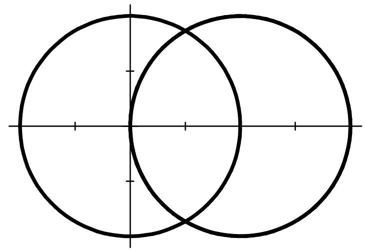
b. $\frac{2\pi}{9}$

c. $\frac{1}{3}$

d. $\frac{2}{3}$

e. $\frac{2\pi}{3}$

3. Find the area inside the circle $r = 2\cos\theta$ but outside the circle $r = 1$.



a. $\frac{\pi}{3} - \cos\frac{\pi}{3}$

b. $\frac{2\pi}{3} + \cos\frac{2\pi}{3}$

c. $\frac{\pi}{3} + \sin\frac{2\pi}{3}$

d. $\frac{2\pi}{3} - \sin\frac{\pi}{3}$

e. $\frac{2\pi}{3} + \sin\frac{\pi}{3}$

4. Compute $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx$. HINT: Switch to polar coordinates.

a. $\frac{\pi}{2} e^2$

b. $\frac{\pi}{2} e^4$

c. $\frac{\pi}{2}(e^2 - 1)$

d. $\frac{\pi}{2}(e^4 - 1)$

e. $\frac{\pi}{4}(e^4 - 1)$

5. Compute the mass of the solid cone $\sqrt{x^2+y^2} \le z \le 4$ if the volume density is $\rho = z$.

a. 4π

b. 8π

c. 16π

d. 32π

e. 64π

6. Compute the center of mass of the solid cone $\sqrt{x^2+y^2} \le z \le 4$ if the volume density is $\rho = z$.

a. $\left(0, 0, \frac{5}{16}\right)$

b. $\left(0, 0, \frac{16}{5}\right)$

c. $\left(0, 0, \frac{1024\pi}{5}\right)$

d. $\left(0, 0, \frac{5}{1024\pi}\right)$

e. $\left(0, 0, \frac{8}{5}\right)$

7. Find the average value of the function $f = \dfrac{1}{x^2+y^2+z^2}$ over the solid region between the two spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$. HINT: $f_{\text{ave}} = \dfrac{\iiint f\,dV}{\iiint 1\,dV}$

a. $\frac{3}{4}$

b. $\frac{5}{8}$

c. $\frac{3}{7}$

d. 4π

e. 8π

8. Compute $\iint \cos\left(\frac{x^2}{9} + \frac{y^2}{4}\right) dx\,dy$ over the region inside the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

HINT: Elliptical coordinates are $x = 3t\cos\theta$, $y = 2t\sin\theta$.

a. $6\pi\sin(1) - 6\pi$

b. $6\pi\sin(1)$

c. $6\pi - 6\pi\cos(1)$

d. $-6\pi\cos(1)$

e. $6\pi\cos(1)$

9. Compute $\int_{(0,0)}^{(\sqrt{2},2)} x\,ds$ along the parabola $y = x^2$ parametrized as $\vec{r}(t) = (t, t^2)$.

a. $\frac{13}{6}$

b. $\frac{9}{4}$

c. $\frac{27}{4}$

d. $\frac{1}{12}(17^{3/2} - 1)$

e. $\frac{17^{3/2}}{12}$

10. Compute $\int_{(0,0)}^{(\sqrt{2},2)} \vec{\nabla} f \cdot d\vec{s}$ for $f = xy$ along the parabola $y = x^2$ parametrized as $\vec{r}(t) = (t, t^2)$.

a. 2

b. $2\sqrt{2}$

c. 4

d. $4\sqrt{2}$

e. 8

11. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-yz^2, xz^2, z^3)$ counterclockwise around the circle $x^2 + y^2 = 4$ with $z = 4$. HINT: Parametrize the circle.

a. 64

b. 128

c. 64π

d. 128π

e. 256π

12. Compute $\iiint \vec{\nabla} \cdot \vec{F}\, dV$ for $\vec{F} = (xy^2, yz^2, zx^2)$ over the solid hemisphere $x^2 + y^2 + z^2 \le 25$ with $z \ge 0$.

a. $\frac{125}{3}\pi$

b. $\frac{250}{3}\pi$

c. $\frac{500}{3}\pi$

d. 1250π

e. 2500π

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (15 points) Find the volume of the largest box such the length plus twice the width plus three times the height is 36.

14. (10 points) Find the area of the surface $\vec{R}(p,q) = (p, p+q^2, p-q^2)$ for $0 \le p \le 3$ and $0 \le q \le 2$.

$\vec{e}_p =$

$\vec{e}_q =$

$\vec{N} =$

$\left|\vec{N}\right| =$

$A =$

15. (15 points) Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-yz^2, xz^2, z^3)$ over the paraboloid $z = x^2 + y^2$ with $z \le 4$, oriented **up and in**, and parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

$\vec{\nabla} \times \vec{F} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

16. (5 points) Mark the Project that you worked on and then answer the questions in 1 or 2 sentences.

___Cauchy's Gradient Method
How did the procedure for finding a maximum differ from the procedure for finding a minimum?

___Seeing a Blimp
Approximately how many degrees wide was the blimp? (1 digit accuracy)

___The Trash Dumpster
Draw the basic shape of your dumpster. Label your variables. Label your hinges and welds.

___Locating an Apartment
What was the geometrical condition satisfied by the location of the apartment?
A diagram may help.

___Rectangles and Triangles
Where were the lines in the rectangle which minimized the sum of the areas?
Where were the lines in the triangle which minimized the sum of the areas?
Two diagrams may help.

___Exact Gradient Method
What is the relation between the velocity of your curve and the gradient of the function to find a maximum? What is the relation to find a minimum?