Name____________________________

1-11	/44	14	/10
12	/15	15	/20
13	/15	Total	/104

MATH 253 Final Fall 2009

Section 501,503 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. Find the point where the line $x = 1+2t$, $y = 8-3t$, $z = 2-2t$ intersects the plane $x-y+z = 1$. At this point $x+y+z =$

a. 9

b. 5

c. 2

d. 1

e. 0

2. Find the plane tangent to the graph of $z = \cos(x+2y)$ at the point $\left(\frac{\pi}{6}, \frac{\pi}{6}\right)$. The z-intercept is

a. 0

b. $\frac{\pi}{6}$

c. $\frac{\pi}{3}$

d. $\frac{\pi}{2}$

e. π

3. Find the plane tangent to the surface $\frac{x}{z} + \frac{z}{y} = 5$ at the point $P = (6,1,3)$. The z-intercept is

a. $(0,0,0)$

b. $(0,0,-5)$

c. $(0,0,5)$

d. $(0,0,-10)$

e. $(0,0,10)$

4. A circuit has two resistors $R_1 = 200\ \Omega$ and $R_2 = 300\ \Omega$ in parallel. The net resistance R satisfies $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 is increasing at $2\ \Omega$/sec and R_2 is decreasing at $9\ \Omega$/sec at what rate is R changing?

a. $\frac{9}{50}\ \Omega$/sec

b. $\frac{18}{25}\ \Omega$/sec

c. $-\frac{9}{50}\ \Omega$/sec

d. $-\frac{9}{25}\ \Omega$/sec

e. $-\frac{18}{25}\ \Omega$/sec

5. Ham Duet is flying the Millenium Eagle through a galactic dust storm. Currently, his position is $P = (10,-20,30)$ and his velocity is $\vec{v} = (4,-12,3)$. He measures that currently the dust density is $\rho = 500$ and its gradient is $\vec{\nabla}\rho = (-2,1,2)$. Find the current rate of change of the dust density as seen by Ham.

a. 514

b. 486

c. 28

d. 14

e. -14

6. Under the same conditions as in #5, in what **unit** vector direction should Ham travel to **decrease** the dust density as quickly as possible?

a. $(-2,1,2)$

b. $(2,-1,-2)$

c. $\left(\frac{2}{3},\frac{-1}{3},\frac{-2}{3}\right)$

d. $\left(\frac{4}{13},\frac{-12}{13},\frac{3}{13}\right)$

e. $\left(\frac{-4}{13},\frac{12}{13},\frac{-3}{13}\right)$

7. The point $(1,-2)$ is a critical point of the function $f = x^2y^2 + \frac{8}{x} - \frac{16}{y}$. Use the Second Derivative Test to classify the point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

8. Compute $\oint \vec{F} \cdot d\vec{s}$ counterclockwise around the circle $x^2 + y^2 = 4$ for $\vec{F} = (x^4 - y^3, y^4 + x^3)$.

HINT: Use the Fundamental Theorem of Calculus for Curves or Green's Theorem.

a. 0

b. 8π

c. 16π

d. 24π

e. 32π

9. The surface of an apple A may be given in spherical coordinates by $\rho = 1 - \cos\varphi$ and may be parametrized by $R(\phi,\theta) = ((1-\cos\varphi)\sin\varphi\cos\theta, (1-\cos\varphi)\sin\varphi\sin\theta, (1-\cos\varphi)\cos\varphi)$.

Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the apple with outward normal for $\vec{F} = (xyz^2, yzx^2, zxy^2)$.

HINT: Use Stokes' Theorem or Gauss' Theorem.

a. 0

b. 4π

c. 12π

d. $\frac{32}{3}\pi$

e. $\frac{64}{3}\pi$

10. Find the mass of the spiral $\vec{r}(\theta) = (\theta\cos\theta, \theta\sin\theta)$ for $0 \le \theta \le 6\pi$ if the linear density is $\rho = \sqrt{x^2 + y^2}$.

a. $\frac{1}{2}\ln\left(6\pi + \sqrt{1 + 36\pi^2}\right) + 3\pi\sqrt{1 + 36\pi^2}$

b. $\frac{1}{2}\ln\left(6\pi + \sqrt{1 + 6\pi}\right) - 3\pi\sqrt{1 + 6\pi}$

c. $\frac{1}{2}\ln\left(6\pi + \sqrt{1 + 6\pi}\right) + 3\pi\sqrt{1 + 6\pi}$

d. $\frac{1}{3}(1 + 36\pi^2)^{3/2} - \frac{1}{3}$

e. $\frac{1}{3}(1 + 6\pi)^{3/2} - \frac{1}{3}$

11. Use Stokes' Theorem to compute $\oint \vec{F} \cdot d\vec{s}$ around the triangle with vertices $A = (2,0,0)$, $B = (0,3,0)$ and $C = (0,0,6)$, traversed from A to B to C to A for $\vec{F} = (y,z,x)$.

Note: The plane of the triangle may be parametrized as $\vec{R}(x,y) = (x,y,6 - 3x - 2y)$.

a. -24

b. -18

c. 12

d. 18

e. 24

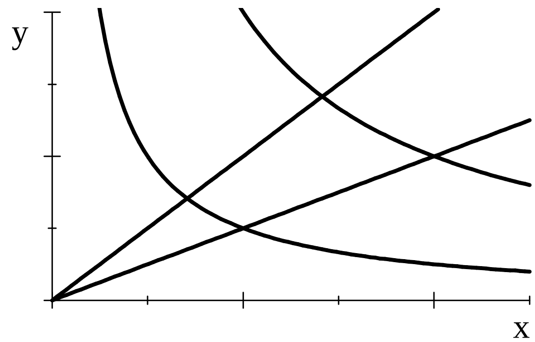
Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 points) Compute $\iint_D y^2\,dx\,dy$ over the "diamond shaped" region D in the first quadrant bounded by the hyperbolas

$y = \frac{1}{x}$ and $y = \frac{4}{x}$

and the lines

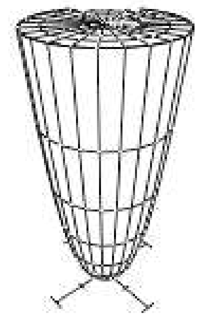
$y = x$ and $y = 2x$



HINT: Use the coordinates $u = xy$, $v = \frac{y}{x}$. Solve for x and y.

13. (15 points) Find the volume and z-component of the centroid (center of mass with $\rho = 1$) of the solid between the surfaces

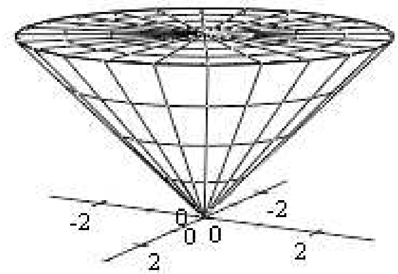
$$z = (x^2 + y^2)^{3/2} \quad \text{and} \quad z = 8.$$



14. (10 points) Find the point in the first octant on the graph of $xy^2z^4 = 32$ which is closest to the origin.

HINTS: What is the square of the distance from a point to the origin? Lagrange multipliers are easier.

15. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (xy^2, yx^2, z^3)$ and the volume above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$. Use the following steps:



a. Compute the volume integral:

$\vec{\nabla} \cdot \vec{F} =$

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

b. Compute the surface integral over the disk using the parametrization $\vec{R}(r,\theta) = (\ \ r\cos\theta\ \ ,\ \ r\sin\theta\ \ ,\ \ 2\ \ )$:

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(r,\theta)\right) =$

$\iint\limits_D \vec{F} \cdot d\vec{S} =$

Continued

Recall: $\vec{F} = (xy^2, yx^2, z^3)$

c. Compute the surface integral over the cone using the parametrization

$\vec{R}(r,\theta) = (\quad r\cos\theta \quad , \quad r\sin\theta \quad , \quad r \quad)$:

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(r,\theta)\right) =$

$\iint_C \vec{F} \cdot d\vec{S} =$

d. Compute the surface integral over the total boundary:

$\iint_{\partial V} \vec{F} \cdot d\vec{S} =$