Name________________________________ID________________Section________________

MATH 253 EXAM 2 Spring 1998

Sections 501-503 P. Yasskin

1. (5 points) Compute $\int_0^1 \int_{x^2}^x x^2 y \; dy \; dx$.

a. $\frac{1}{70}$

b. $\frac{1}{35}$

c. $\frac{2}{35}$

d. $\frac{1}{14}$

e. $\frac{1}{7}$

2. (5 points) Find the volume below the plane $z = 4x + 10y$ above the region between the parabola $y = x^2$ and the line $y = x$.

a. 1

b. 2

c. 3

d. 4

e. 5

3. (5 points) Compute $\int_0^1 \int_0^x \int_0^{x+y} x \; dz \; dy \; dx$.

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{3}{8}$

d. $\frac{1}{2}$

e. $\frac{5}{8}$

4. (5 points) Compute $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2) \; dx \; dy$.

a. 1

b. 2

c. 3

d. 4

e. Cannot be computed.

5. (5 points) Reversing the order of integration gives $\int_0^1 \int_{x^2}^{x} f(x,y)\ dy\ dx =$

a. $\int_0^1 \int_{x^2}^{x} f(x,y)\ dx\ dy$

b. $\int_{x^2}^{x} \int_0^1 f(x,y)\ dx\ dy$

c. $\int_0^1 \int_{y}^{\sqrt{y}} f(x,y)\ dx\ dy$

d. $\int_0^1 \int_{y^2}^{y} f(x,y)\ dx\ dy$

e. $\int_0^1 \int_{\sqrt{y}}^{y} f(x,y)\ dx\ dy$

6. (5 points) Compute $\iint\limits_D e^{-x^2-y^2}\ dx\ dy$. over the disk $D = \{(x,y)\ |\ x^2+y^2 \leq 4\}$.

a. $\frac{\pi}{2}(e^4-1)$

b. $\frac{\pi}{2}(1-e^{-4})$

c. $\pi(e^4-1)$

d. $\pi(1-e^{-4})$

e. Cannot be computed.

7. (5 points) Find the area of one loop of the rose $r = \sin(3\theta)$.

a. $\frac{\pi}{12} + \frac{\sqrt{3}}{48}$

b. $\frac{\pi}{12} - \frac{\sqrt{3}}{48}$

c. $\frac{\pi}{12} + \frac{1}{24}$

d. $\frac{\pi}{12} - \frac{1}{24}$

e. $\frac{\pi}{12}$

8. (5 points) Find the mass of the cylinder $x^2+y^2 \leq 4$ for $0 \leq z \leq 3$ if the density is $\rho = x^2+y^2+z^2$.

a. 24π

b. 30π

c. 36π

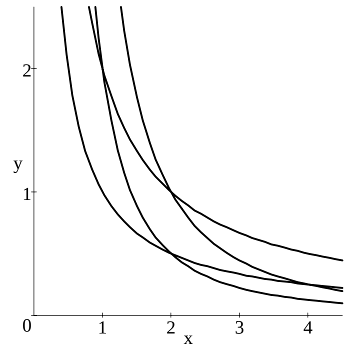
d. 52π

e. 60π

9. (20 points) Find the mass M and center of mass $(\bar{x}, \bar{y})$ of the quarter of the circle $x^2 + y^2 \leq 4$ in the first quadrant if the density is $\rho = 3 + x^2 + y^2$.
HINT: By symmetry, $\bar{x} = \bar{y}$. So you only need to compute $\bar{x}$.

10. (20 points) Compute $\iint_R x^2y \; dx\,dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{2}{x}, \qquad y = \frac{2}{x^2}, \qquad y = \frac{4}{x^2}$$



FULL CREDIT for integrating in the curvilinear coordinates $u = xy$ and $v = x^2y$. (Solve for x and y.)

HALF CREDIT for integrating in rectangular coordinates.

11. (20 points) Find the volume V and the z-component of the centroid $\bar{z}$ of the hemisphere $0 \le z \le \sqrt{9 - x^2 - y^2}$.

1-8	
9	
10	
11	