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MATH 253 EXAM 3 Spring 1998

Sections 501-503 P. Yasskin

Multiple Choice: (5 points each)

1. If $F = (xy, yz, xz)$ then $\vec{\nabla} \bullet \vec{F} =$

a. $y - z + x$
b. $(-y, z, -x)$
c. $x + y + z$
d. $(-y, -z, -x)$
e. $-x + y - z$

2. If $F = (xy, yz, xz)$ then $\vec{\nabla} \times \vec{F} =$

a. $y - z + x$
b. $(-y, z, -x)$
c. $x + y + z$
d. $(-y, -z, -x)$
e. $-x + y - z$

3. Compute the line integral $\int y\,dx - x\,dy$ counterclockwise around the semicircle $x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$. (HINT: Parametrize the curve.)

a. -4π
b. -2π
c. π
d. 2π
e. 4π

4. Compute the line integral $\int \vec{F} \bullet d\vec{s}$ for the vector field $\vec{F} = \left(\frac{1}{x}, \frac{1}{y}\right)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)}\right)$ for $0 \le t \le \sqrt{\pi}$. (HINT: Find a potential.)

a. -2
b. 0
c. $\frac{2}{e}$
d. 1
e. π

5. Compute $\oint (5x+3y)\,dx + (x-2y)\,dy$ counterclockwise around the edge of the rectangle $1 \le x \le 5,\ \ 3 \le y \le 6$. (HINT: Use Green's Theorem.)

a. 36
b. 24
c. 12
d. -24
e. -36

6. Compute $\iint\limits_{\partial C} \vec{F} \bullet d\vec{S}$ for the vector field $\vec{F} = \left(zx^3, zy^3, z^2(x^2+y^2)\right)$ over the complete surface of the solid cylinder $C = \left\{(x,y,z)\ \middle|\ x^2+y^2 \le 4,\ 0 \le z \le 3\right\}$.

a. 360π
b. 180π
c. 90π
d. 60π
e. 30π

7. If $f(x,y,z) = x\sin(yz) - y\cos(xz) + z\tan(xy)$ then $\vec{\nabla} \times \vec{\nabla} f =$

a. $z\sin(yz) + z\cos(xz) + xy\sec^2(xy)$
b. $\sin(yz) - \cos(xz) + \tan(xy)$
c. $\cos(yz) + \sin(xz) + \sec^2(xy)$
d. 0
e. Does not exist.

8. (25 points) Green's Theorem states that if R is a nice region in the plane and ∂R is its boundary curve traversed counterclockwise then

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint_{\partial R} P\,dx + Q\,dy$$

Verify Green's Theorem if $P = -y^3$ and $Q = x^3$ and R is the region inside the circle $x^2 + y^2 = 9$.

8a. (5 points) Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$. (HINT: Use rectangular coordinates.)

8b. (10 points) Compute $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$.

a. (HINT: Switch to polar coordinates and don't forget the Jacobian.)

8c. (10 points) Compute $\oint_{\partial R} P\,dx + Q\,dy$. (HINT: Parametrize of the boundary circle.)

9. (30 points) Stokes' Theorem states that if S is a surface in 3-space and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S then

$$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$$

Verify Stokes' Theorem if $F = (-yx^2, xy^2, x^2 + y^2)$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ below $z = 2$ with normal pointing in and up.

9a. (5 points) Compute $\vec{\nabla} \times \vec{F}$. (HINT: Use rectangular coordinates.)

9b. (10 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$.

a. (HINT: Here is the parametrization of the cone and the steps you should use. Remember to check the orientation of the surface.)

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

9c. (15 points) Compute $\oint_{\partial S} \vec{F} \bullet d\vec{s}$. Recall $F = (-yx^2, xy^2, x^2 + y^2)$.

(HINT: Parametrize of the boundary circle. Remember to check the orientation of the curve.)

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial S} \vec{F} \bullet d\vec{s} =$

10. (10 points)

The spider web at the right is the graph of the hyperbolic paraboloid $z = xy$.
It may be parametrized as
$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2\sin\theta\cos\theta)$.
Find the area of the web for $r \le \sqrt{8}$.