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MATH 253 EXAM 2 Fall 1998

Sections 501-503 P. Yasskin

Multiple Choice: (5 points each)

1. Compute $\int_1^2 \int_1^x y\, dy\, dx$.

a. $-\frac{1}{3}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{7}{6}$
e. $\frac{4}{3}$

2. Find the volume under the surface $z = 2x^2y$ above the triangle with vertices $(0,0)$, $(1,2)$ and $(0,4)$.

a. $-\frac{1}{3}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{7}{6}$
e. $\frac{4}{3}$

3. Compute $\int_0^1 \int_{\sqrt{y}}^1 \int_0^y x\ y\ dz\ dx\ dy.$

a. $\frac{1}{24}$

b. $\frac{1}{12}$

c. $\frac{1}{2\sqrt{2}}$

d. $\frac{3}{2\sqrt{2}}$

e. $3\sqrt{2}$

4. Find the area enclosed by one loop of the daisy $r = \sin 4\theta$:

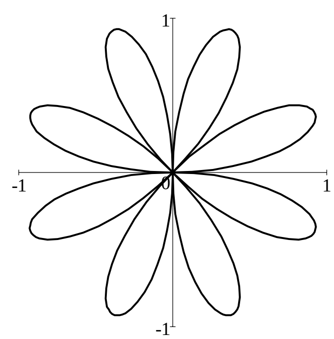
a. $\frac{\pi}{32}$

b. $\frac{\pi}{16}$

c. $\frac{\pi}{8}$

d. $\frac{\pi}{4}$

e. $\frac{\pi}{2}$



5. Compute $\int_0^{\sqrt[4]{\pi}} \int_{y^2}^{\sqrt{\pi}} y \sin(x^2)\ dx\ dy$.

a. $\frac{1}{4}$

b. $\frac{1}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{2}$

e. $\frac{3}{4}\pi\sqrt{\pi}$

6. Compute $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\ dx\ dy$ by converting to polar coordinates.

a. $\frac{\pi}{4}\ln 3$

b. $\frac{\pi}{8}\ln 3$

c. $\frac{\pi}{4}\ln 5$

d. $\frac{\pi}{8}\ln 5$

e. $\frac{\pi}{4}\ln 17$

7. Compute $\iiint_D \sqrt{x^2+y^2}\, dV$ over the region D bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

a. $\frac{4\pi}{5}3^4$

b. $\frac{\pi}{2}3^4$

c. $\frac{\pi}{2}3^5$

d. $2\pi 3^4$

e. $2\pi 3^5$

8. Compute $\iiint_H z\, dV$ over the solid hemisphere H below the sphere $x^2+y^2+z^2 = 4$ and above the xy-plane.

a. π

b. 2π

c. 4π

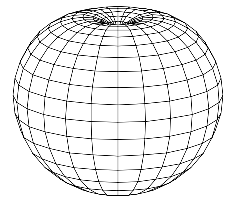
d. $\frac{4\pi}{3}$

e. $\frac{8\pi}{3}$

9. (20 points) Find the mass M and center of mass $(\bar{x}, \bar{y})$ of the region above the parabola $y = x^2$ below the line $y = 4$, if the density is $\rho = y$. (12 points for setup.)
HINT: By symmetry, $\bar{x} = 0$. So you only need to compute $\bar{y}$.

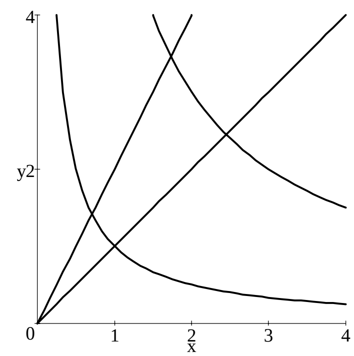
10. (10 points) Plot the polar curve $r = 1 - \cos\theta$. Show your work.

11. (20 points) Find the volume V and the z-component of the centroid $\bar{z}$ of the apple given in spherical coordinates by $\rho = 1 - \cos\phi$. (16 points for setup.)
Hint: In the ϕ-integral, use the substitution $u = 1 - \cos\phi$.



12. (10 points) Compute $\iint_R y^2 \, dx\, dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{6}{x}, \qquad y = x, \qquad y = 2x$$



FULL CREDIT for integrating in the curvilinear coordinates (u,v) where $u^2 = xy$ and $v^2 = \frac{y}{x}$. (Solve for x and y.)

HALF CREDIT for integrating in rectangular coordinates.