1-12	/120
13	/25
14	/15
15	/30
16	/20

Name____________________ ID____________ Section______

MATH 253 FINAL EXAM Fall 1998

Sections 501-503 P. Yasskin

Multiple Choice: (10 points each)

HINTS: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

1. Compute $\lim_{n\to\infty} \frac{3n^2}{1+n^3}$

- **a.** 0
- **b.** 1
- **c.** 2
- **d.** 3
- **e.** Divergent

2. Find r such that $5 + 5r + 5r^2 + 5r^3 + 5r^4 + \cdots = 3$.

- **a.** $\frac{2}{5}$
- **b.** $-\frac{2}{5}$
- **c.** $\frac{3}{5}$
- **d.** $\frac{5}{3}$
- **e.** $-\frac{2}{3}$

3. Compute $\sum_{k=1}^{99} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \right)$

- **a.** .9
- **b.** .99
- **c.** 1
- **d.** 1.1
- **e.** Divergent

4. The series $\sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}$ is

a. divergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

b. convergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

c. divergent by the ratio test.

d. convergent by the ratio test.

e. divergent by the n^{th}-term test.

5. The series $\sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{1 + n^3}$ is

a. absolutely convergent.

b. conditionally convergent.

c. divergent to ∞.

d. divergent to $-\infty$.

e. oscillatory divergent.

6. Compute $\lim_{x \to 0} \frac{\cos(2x) - 1 + 2x^2}{x^4}$

a. 0

b. $\frac{1}{24}$

c. $\frac{1}{12}$

d. $\frac{2}{3}$

e. ∞

7. Given that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ (for $|x| < 1$), then (for $|x| < 1$) we have $\sum_{n=0}^{\infty} n x^n =$

a. $\frac{1}{1-x}$

b. $\frac{1}{(1-x)^2}$

c. $\frac{x}{(1-x)^2}$

d. $\frac{x}{1-x}$

e. $\frac{n}{1-x}$

8. Find the x-coordinate of the center of mass of the rectangle $0 \le x \le 3$, $0 \le y \le 2$ if the density is $\rho = xy$.

a. $.5$
b. 1
c. 1.5
d. 2
e. 2.5

9. Find the mass of the solid inside the cylinder $x^2 + y^2 = 1$ above the paraboloid $z = x^2 + y^2$ and below the plane $z = 2$ if the density is $\rho = x^2 + y^2$.

a. $\frac{\pi}{2}$
b. $\frac{2\pi}{3}$
c. π
d. $\frac{3\pi}{2}$
e. 2π

10. If $F = (xz^2, -yz^2, z^3)$ then $\vec{\nabla} \bullet \vec{F} =$

a. $(z^2, z^2, 3z^2)$
b. $(z^2, -z^2, 3z^2)$
c. $3z^2$
d. $5z^2$
e. $2(y - x)z$

11. If $F = (xz^2, -yz^2, z^3)$ then $\vec{\nabla} \times \vec{F} =$

a. $2(y - x)z$
b. $2(x + y)z$
c. $(2yz, -2xz, 0)$
d. $(2yz, 2xz, 0)$
e. $\vec{0}$

12. Compute $\int_{\vec{r}(t)} y^2 e^{xy}\,dx + (1+xy)e^{xy}\,dy$ along the spiral $\vec{r}(t) = (t\cos t, t\sin t)$ from $t = \pi$ to $t = 3\pi$.

HINT: Find a scalar potential for $\vec{F} = (y^2 e^{xy}, (1+xy)e^{xy})$.

a. -2π

b. 0

c. π

d. 3π

e. 4π

Work-Out Problems

13. (25 points) You are given: $e^{(-x^2)} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \cdots$.

a. (10 pt) If $f(x) = e^{(-x^2)}$, find $f^{(6)}(0)$.

b. (10 pt) Use the **quadratic** Taylor polynomial approximation about $x = 0$ for $e^{(-x^2)}$ to estimate $\int_0^{0.1} e^{(-x^2)}\,dx$. (Keep 8 digits.)

c. (5 pt) Your result in (b) is equal to $\int_0^{0.1} e^{(-x^2)}\,dx$ to within $\pm$ how much? Why?

14. (15 points) Find the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{3^n n^3}$.

Be sure to identify each of the following and give reasons:

(1 pt) Center of Convergence: $a =$ ____________

Radius of Convergence: $R =$ ____________ (5 pt)

(1 pt) Right Endpoint: $x =$ ____________

At the Right Endpoint the Series { Converges / Diverges } (circle one) (3 pt)

(1 pt) Left Endpoint: $x =$ ____________

At the Left Endpoint the Series { Converges / Diverges } (circle one) (3 pt)

(1 pt) Interval of Convergence: ________________________

15. (30 points) Stokes' Theorem states that if S is a surface in 3-space and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S then

$$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$$

Verify Stokes' Theorem if $F = (-yz, xz, z^2)$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ below $z = 2$ with normal pointing in and up.

a. (5 pt) Compute $\vec{\nabla} \times \vec{F}$. (HINT: Use rectangular coordinates.)

b. (10 pt) Compute $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$.

(HINT: Here is the parametrization of the cone and the steps you should use. Remember to check the orientation of the surface.)

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

15c. (15 pt) Compute $\oint_{\partial S} \vec{F} \bullet d\vec{s}$. Recall $F = (-yz, xz, z^2)$.

(HINT: Parametrize the boundary circle. Remember to check the orientation of the curve.)

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial S} \vec{F} \bullet d\vec{s} =$

16. (20 points) Find the minimum value and its location(s) for the function $f(x, y) = xy$ on the ellipse $9x^2 + 4y^2 = 72$.