Name____________________ ID___________ Section_____

1-5	/40	8	/20
6	/10	9	/10
7	/10	10	/10

MATH 253 Honors   Final Exam   Fall 2002

Sections 201-202   P. Yasskin

Multiple Choice: (8 points each)   Work Out: (points indicated)

1. Find the volume of the parallelepiped with edges $\vec{u} = (1,0,3)$, $\vec{v} = (0,2,-1)$ and $\vec{w} = (2,0,2)$.

a. -8
b. -4
c. 4
d. 8
e. 16

2. Duke Skywater is flying the Millenium Eagle through a polaron field. His galactic coordinates are $(2300,4200,1600)$ measured in lightseconds and his velocity is $\vec{v} = (.2,.3,.4)$ measured in lightseconds per second. He measures the strength of the polaron field is $p = 274$ milliwookies and its gradient is $\vec{\nabla}p = (3,2,2)$ milliwookies per lightsecond. Assuming a linear approximation for the polaron field and that his velocity is constant, how many seconds will Duke need to wait until the polaron field has grown to 286 milliwookies?

a. 2
b. 3
c. 4
d. 6
e. 12

3. Find the plane tangent to the hyperbolic paraboloid $x = yz$ at the point $P = (6, 3, 2)$. Which of the following points does **not** lie on this plane?

a. $(-6, 0, 0)$
b. $(0, 3, 0)$
c. $(0, 0, 2)$
d. $(-1, 1, 1)$
e. $(1, -1, -1)$

4. A airplane is circling with constant speed above Kyle Field along the curve $\vec{r}(t) = (\cos(8\pi t), \sin(8\pi t), 2)$ where distances are in miles and time is in hours. Find the tangential acceleration a_T, where the acceleration is $\vec{a} = a_T\hat{T} + a_N\hat{N}$.

a. 0
b. 8π
c. -8π
d. $64\pi^2$
e. $-64\pi^2$

5. Find the volume below the plane $z = 6 - 2y$ above the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 3, 0)$.

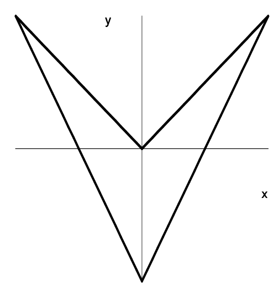
a. 3
b. 6
c. 9
d. 12
e. 15

6. (10 points) Find the location and value of the minimum of the function $f(x,y,z) = x^2 + 2y^2 + 3z^2$ on the plane $x + y + z = 11$.

7. (10 points) Consider the region between the curves

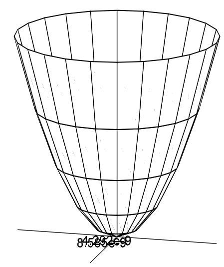
$$y = 2|x| - 2 \quad \text{and} \quad y = |x|.$$

If the density is $\delta = 2 + 2y$ compute the mass and y-component of the center of mass of this region.
(7 points for setup. 3 points for evaluation.)



8. $\left(20 \text{ points}\right)$ **Stokes' Theorem** states that if S is a nice surface in $\mathbf{R}^3$ and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S and $\vec{F}$ is a nice vector field on S then

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$



Verify Stokes' Theorem if

$F = (y, -x, x^2 + y^2)$

and S is the paraboloid $z = x^2 + y^2$ for $z \le 4$

with **normal pointing up and in**.

Remember to check the orientations.

The paraboloid may be parametrized by:

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$$

a. (10) Compute $\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ using the following steps:

$\vec{\nabla} \times \vec{F} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

b. (10) Recall $F = (y, -x, x^2 + y^2)$ and S is the paraboloid $z = x^2 + y^2$ for $z \le 4$ with **normal pointing up and in**. Compute $\oint_{\partial S} \vec{F} \cdot d\vec{s}$ using the following steps:

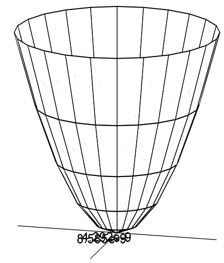
$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial S} \vec{F} \cdot d\vec{s} =$

9. (10 points) The paraboloid at the right is the graph of the equation $z = x^2 + y^2$.



It may be parametrized as

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2).$$

Find the area of the paraboloid for $z \le 4$.

HINT: Use results from #8.

10. (10 points) A paraboloid in $\mathbf{R}^4$ with coordinates (w,x,y,z), may be parametrized by

$$(w,x,y,z) = \vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2, r^2) \quad \text{for} \quad 0 \le r \le 3 \quad \text{and} \quad 0 \le \theta \le 2\pi.$$

Compute $I = \iint (xz\,dw\,dy - wy\,dx\,dz)$ over the surface.