Name________________________ ID__________________

1-11	/55	14	/12
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Total	/103		

MATH 253 Exam 1 Fall 2006

Sections 201,202 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. The vertices of a triangle are $P = (3,4,-5)$, $Q = (3,5,-4)$ and $R = (5,2,-5)$.
Find the angle at P.

a. $90°$

b. $120°$

c. $135°$

d. $150°$

e. $180°$

2. Find the volume of the parallelepiped with edge vectors:
$\vec{a} = \langle 4,1,2\rangle$ $\vec{b} = \langle 2,2,1\rangle$ $\vec{c} = \langle 1,3,0\rangle$

a. -3

b. 0

c. $\sqrt{3}$

d. 3

e. 9

3. Consider the set of all points P whose distance from $(1,0,0)$ is 3 times its distance from $(-1,0,0)$. This set is a

a. sphere.

b. ellipsoid.

c. hyperboloid.

d. elliptic paraboloid.

e. hyperbolic paraboloid.

4. For the curve $\vec{r}(t) = (\sin^2 t, \quad \cos^2 t, \quad \sin^2 t - \cos^2 t)$ which of the following is FALSE?

a. $\vec{v} = \langle 2\sin t\cos t, \quad -2\sin t\cos t, \quad 4\sin t\cos t\rangle$

b. $|\vec{v}| = \sqrt{24}\,\sin t\cos t$

c. $\hat{T} = \left\langle \frac{2}{\sqrt{24}}, \quad \frac{-2}{\sqrt{24}}, \quad \frac{4}{\sqrt{24}} \right\rangle$

d. $a_T = 0$

e. $a_N = 0$

5. For the curve $\vec{r}(t) = (\sin^2 t, \quad \cos^2 t, \quad \sin^2 t - \cos^2 t)$ compute the arc length between $\vec{r}(0) = (0, \quad 1, \quad -1)$ and $\vec{r}\left(\frac{\pi}{2}\right) = (1, \quad 0, \quad 1)$.

a. $\frac{1}{4}\sqrt{6}$

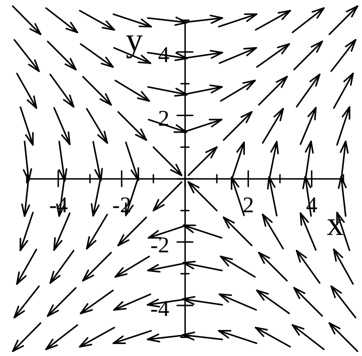
b. $\frac{1}{2}\sqrt{6}$

c. $\sqrt{6}$

d. $2\sqrt{6}$

e. 4

6. The plot at the right represents which vector field?



a. $\vec{A} = \langle x, y \rangle$

b. $\vec{B} = \left\langle \dfrac{x}{\sqrt{x^2+y^2}}, \dfrac{y}{\sqrt{x^2+y^2}} \right\rangle$

c. $\vec{C} = \langle y, x \rangle$

d. $\vec{D} = \left\langle \dfrac{y}{\sqrt{x^2+y^2}}, \dfrac{x}{\sqrt{x^2+y^2}} \right\rangle$

e. $\vec{E} = \langle x+y, x-y \rangle$

7. Describe the level surfaces of $f(x,y,z) = x^2 - y^2 - z^2$.

a. Elliptic Paraboloids

b. Elliptic and Hyperbolic Paraboloids

c. Hyperboloids of 1-sheet only

d. Hyperboloids of 2-sheets only

e. Hyperboloids of 1-sheet or 2-sheets

8. Find the plane tangent to the graph of $z = xe^{xy}$ at the point $(2,0)$. Its z-intercept is

a. 0

b. 2

c. -2

d. 4

e. -4

9. Find the plane tangent to the surface $xyz + z^2 = 28$ at the point $(4, 3, 2)$.

Its z-intercept is

a. 0

b. 5

c. -5

d. 80

e. -80

10. Find the line normal to the surface $xyz + z^2 = 28$ at the point $(4, 3, 2)$.

It intersects the xy-plane at

a. $(4, 3, 2)$

b. $(4, 3, 0)$

c. $\left(\frac{13}{4}, 2, 0\right)$

d. $\left(\frac{19}{4}, 4, 4\right)$

e. $\left(\frac{19}{4}, 4, 0\right)$

11. The salt concentration in a region of sea water is $\rho = xy^2z^3$. A swimmer is located at $(3, 2, 1)$.

In what direction should the swimmer swim to increase the salt concentration as fast as possible?

a. $\langle 4, -12, 36 \rangle$

b. $\langle -4, 12, -36 \rangle$

c. $\langle 4, 12, 36 \rangle$

d. $\langle -4, -12, -36 \rangle$

e. $\langle 4, -12, -36 \rangle$

Work Out: (12 points each. Part credit possible. Show all work.)

Do 4 of the following 5 problems. Cross out the one you do not want graded, here and on page 1. If you do not specify, #12 will be dropped.

12. Which of the following functions satisfy the Laplace equation $f_{xx} + f_{yy} = 0$?

Show your work!

a. $f = x^2 + y^2$

b. $f = x^2 - y^2$

c. $f = x^3 + 3xy^2$

d. $f = x^3 - 3xy^2$

e. $f = e^{-x}\cos y + e^{-y}\cos x$

f. $f = e^{-x}\cos y - e^{-y}\cos x$

13. When two resistors with resistances R_1 and R_2 are connected in parallel, the net resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{or} \qquad R = \frac{R_1 R_2}{R_1 + R_2}.$$

If R_1 and R_2 are measured as $R_1 = 2 \pm 0.01$ ohms and $R_2 = 3 \pm 0.04$ ohms, then R can be calculated as $R = \frac{6}{5} \pm \Delta R$ ohms.

Use differentials to estimate the uncertainty ΔR in the computed value of R.

14. The average of a function f on a curve $\vec{r}(t)$ is $f_{\text{ave}} = \dfrac{\int f\,ds}{\int ds}$.

Find the average of $f(x,y) = x^2$ on the circle $x^2 + y^2 = 9$.

HINTS: Parametrize the circle. $\sin^2 A = \dfrac{1 - \cos(2A)}{2}$ $\cos^2 A = \dfrac{1 + \cos(2A)}{2}$

15. A particle moves along the curve $\vec{r}(t) = (t^3, t^2, t)$ from $(1,1,1)$ to $(8,4,2)$ under the action of the force $\vec{F} = \langle z, y, x \rangle$. Find the work done.

16. The pressure in an ideal gas is given by $P = k\rho T$ where k is a constant, ρ is the density and T is the temperature. At a certain instant, the measuring instruments are located at $r_o = (1,2,3)$ and moving with velocity $\vec{v} = \langle 4,5,6\rangle$ and acceleration $\vec{a} = \langle 7,8,9\rangle$.

At that instant, the density and temperature are measured to be $\rho = 12$ and $T = 300$ and their gradients are $\vec{\nabla}\rho = \langle 0.6, 0.4, 0.2\rangle$ and $\vec{\nabla}T = \langle 2,1,4\rangle$.

Find $\dfrac{dP}{dt}$, the time rate of change of the pressure as seen by the instruments.

Your answer may depend on k.

HINTS: The pressure, P is a function of density, ρ, and temperature, T, which are functions of the position coordinates, (x,y,z), which are functions of time, t. Use the chain rule.