Name____________________________ ID____________________

MATH 253 Final Exam Fall 2006

Sections 201,202 P. Yasskin

1-10	/50
11	/15
12	/15
13	/15
14	/15
Total	/110

Multiple Choice: (5 points each. No part credit.)

1. For the curve $\vec{r}(t) = (t\cos t, t\sin t)$, which of the following is false?

a. The velocity is $\vec{v} = (\cos t - t\sin t, \sin t + t\cos t)$

b. The speed is $|\vec{v}| = \sqrt{1+t^2}$

c. The acceleration is $\vec{a} = (-2\sin t - t\cos t, 2\cos t - t\sin t)$

d. The arclength between $t = 0$ and $t = 1$ is $L = \int_0^1 t\sqrt{1+t^2}\,dt$

e. The tangential acceleration is $a_T = \dfrac{t}{\sqrt{1+t^2}}$

2. Find the line perpendicular to the surface $x^2z^2 + y^4 = 5$ at the point $(2,1,1)$.

a. $(x,y,z) = (1+t, 1+t, 2+2t)$

b. $(x,y,z) = (1+2t, 1+t, 2+t)$

c. $(x,y,z) = (2+t, 1+t, 1+2t)$

d. $(x,y,z) = (1+2t, 1+t, 2+2t)$

e. $(x,y,z) = (2+2t, 1+t, 1+1t)$

3. Let $L = \lim_{(x,y)\to(0,0)} \dfrac{e^{\left(x^2+y^2\right)} - 1}{x^2 + y^2}$

a. L exists and $L = 1$ by looking at the paths $y = mx$.

b. L does not exist by looking at the paths $y = x$ and $y = -x$.

c. L does not exist by looking at polar coordinates.

d. L exists and $L = 0$ by looking at polar coordinates.

e. L exists and $L = 1$ by looking at polar coordinates.

4. The point $(1,-3)$ is a critical point of the function $f = xy^2 - 3x^3 + 6y$. It is a

a. local minimum.

b. local maximum.

c. saddle point.

d. inflection point.

e. The Second Derivative Test fails.

5. Compute the line integral $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (y, x + 2y)$ along the curve $\vec{r}(t) = \left(e^{\sin\left(t^2\right)}, e^{\cos\left(t^2\right)}\right)$ for $0 \le t \le \sqrt{\pi}$. (HINT: Find a scalar potential.)

a. $e^2 + e - \dfrac{1}{e} - \dfrac{1}{e^2}$

b. $\dfrac{1}{e^2} + \dfrac{1}{e} - e - e^2$

c. $e^2 - e + \dfrac{1}{e} - \dfrac{1}{e^2}$

d. $\dfrac{1}{e^2} - \dfrac{1}{e} + e - e^2$

e. 0

6. Compute the line integral $\int y\,dx - x\,dy$ along the curve $y = x^2$ from $(-3,9)$ to $(0,0)$.
HINT: The curve may be parametrized as $r(t) = (t, t^2)$.

a. -9
b. -3
c. 1
d. 3
e. 9

7. Consider the quarter cylinder surface $x^2 + y^2 = 4$ with $x \geq 0$, $y \geq 0$ and $0 \leq z \leq 8$.
Find the total mass of the quarter cylinder surface if the density is $\rho = x$.
The surface may be parametrized by $\vec{R}(\theta, h) = (2\cos\theta, 2\sin\theta, h)$.

a. 32
b. 32π
c. 8
d. 8π
e. 64π

8. Consider the quarter cylinder surface $x^2 + y^2 = 4$ with $x \geq 0$, $y \geq 0$ and $0 \leq z \leq 8$.
Find the y-component of the center of mass of the quarter cylinder if the density is $\rho = x$.

a. $\frac{4}{\pi}$
b. $\frac{\pi}{4}$
c. 32
d. 2
e. 1

9. Compute the line integral $\oint x^2 y\,dx - xy^2\,dy$ counterclockwise around the circle $x^2 + y^2 = 16$. (HINT: Use a theorem.)

a. -128π

b. -64π

c. 0

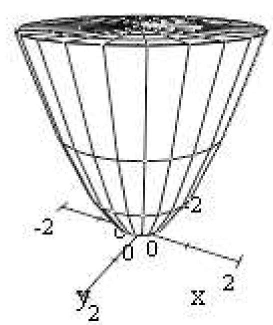
d. 64π

e. 128π

10. Consider the parabolic surface P given by $z = x^2 + y^2$ for $z \le 4$ with normal pointing up and in, the disk D given by $x^2 + y^2 \le 4$ and $z = 4$ with normal pointing up, and the volume V between them. Given that for a certain vector field $\vec{F}$ we have

$$\iiint_V \vec{\nabla} \cdot \vec{F}\,dV = 13 \quad \text{and} \quad \iint_D \vec{F} \cdot d\vec{S} = 4$$

compute $\iint_P \vec{F} \cdot d\vec{S}$.



a. -17

b. -9

c. 5

d. 9

e. 17

Work Out: (15 points each. Part credit possible.)

11. Find the point in the first octant on the graph of $xy^2z^4 = 32$ which is closest to the origin.
You do not need to show it is a maximum. You MUST use the Method of Lagrange Multipliers.
Half credit for the Method of Elminating the Constraint.

12. The hemisphere H given by

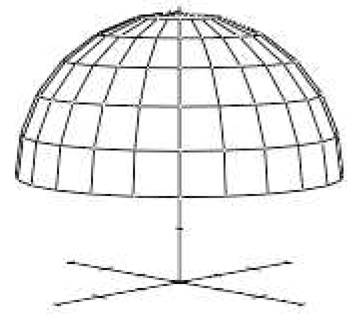
$$x^2 + y^2 + (z-2)^2 = 9 \quad \text{for} \quad z \ge 2$$

has center $(0,0,2)$ and radius 3. Verify Stokes' Theorem

$$\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s}$$

for this hemisphere H with normal pointing up and out

and the vector field $\vec{F} = (yz, -xz, z)$.



Be sure to check and explain the orientations. Use the following steps:

a. The hemisphere may be parametrized by

$$\vec{R}(\theta,\varphi) = (3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 2+3\cos\varphi)$$

Compute the surface integral by successively finding:

$$\vec{e}_\theta,\quad \vec{e}_\varphi,\quad \vec{N},\quad \vec{\nabla}\times\vec{F},\quad \vec{\nabla}\times\vec{F}\left(\vec{R}(\theta,\varphi)\right),\quad \iint_H \vec{\nabla}\times\vec{F}\cdot d\vec{S}$$

Problem Continued

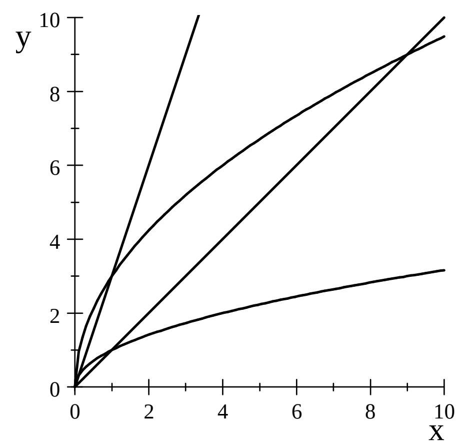
b. Parametrize the boundary circle ∂H and compute the line integral by successively finding:

$\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_{\partial H} \vec{F} \bullet d\vec{s}$. Recall: $\vec{F} = (yz, -xz, z)$

13. Compute $\iint \frac{1}{x^2}\, dx\, dy$ over the diamond shaped region bounded by the curves

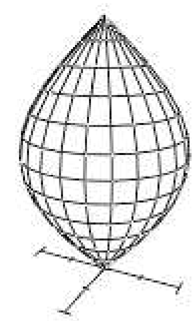
$y = \sqrt{x}$, $y = 3\sqrt{x}$, $y = x$ and $y = 3x$.

HINT: Let $u = \frac{y^2}{x}$ and $v = \frac{y}{x}$.



14. The surface of a football may be approximated in cylindrical coordinates by

$$r = \sin z \qquad \text{for} \quad 0 \le z \le \pi$$



Verify Gauss' Theorem $\displaystyle\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$

for the volume inside the football and the vector field

$$\vec{F} = (2x, 2y, x^2 + y^2)$$

Use the following steps:

a. Compute the volume integral by computing $\vec{\nabla} \cdot \vec{F}$ in rectangular coordinates and then $\displaystyle\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$ in cylindrical coordinates.

b. The surface of the football may be parametrized by $\vec{R}(\theta, h) = (\sin h \cos\theta, \sin h \sin\theta, h)$.
Compute the surface integral by successively finding
$\vec{e}_\theta$, $\vec{e}_h$, $\vec{N}$, $\vec{F}\big(\vec{R}(\theta, h)\big)$, $\vec{F} \cdot \vec{N}$, and $\displaystyle\iint \vec{F} \cdot d\vec{S}$.