1-8	/40
9	/20
10	/20
11	/20
Total	/100

Name______________________________

MATH 253 Exam 2 Fall 2014

Sections 201,202 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^\pi \int_0^\pi \int_0^\varphi \rho^2 \, d\rho \, d\theta \, d\varphi$.

a. $\frac{1}{6}\pi^5$

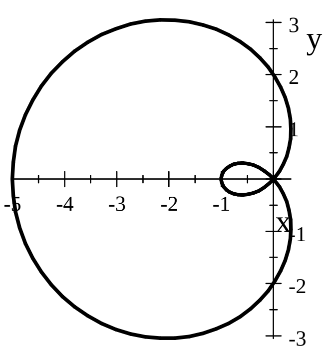
b. $\frac{1}{12}\pi^5$

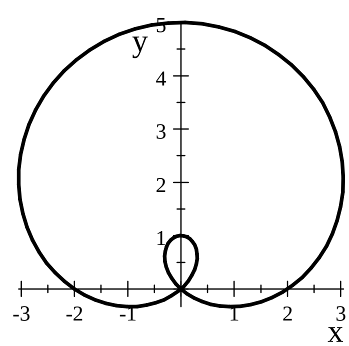
c. $\frac{1}{20}\pi^5$

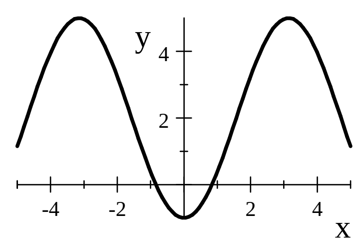
d. $\frac{1}{6}\pi^4$

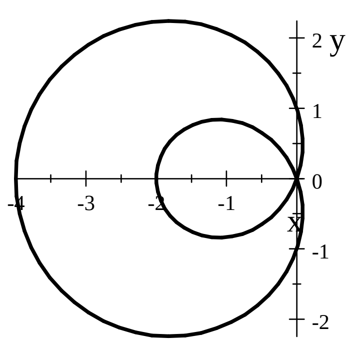
e. $\frac{1}{12}\pi^4$

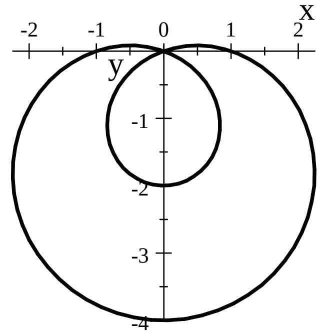
2. Which of the following is the polar plot of $r = 2 - 3\cos(\theta)$?

a.


b.


c.


d.


e.


3. A plate fills the region between the graphs of $x = |y|$ and $x = 4$. Find its mass if its surface density is $\rho = x^2$.

a. $\frac{256}{3}$

b. $\frac{128}{3}$

c. 256

d. 128

e. 64

4. A plate fills the region between the graphs of $x = |y|$ and $x = 4$. Find the x-component of its center of mass if its surface density is $\rho = x^2$.

a. $\frac{16}{5}$

b. $\frac{32}{5}$

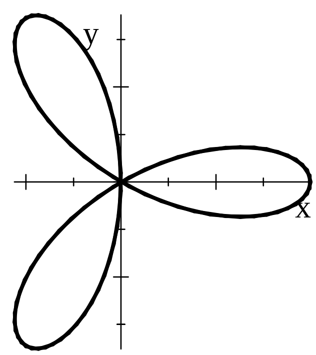
c. $\frac{512}{5}$

d. $\frac{2048}{5}$

e. $\frac{5}{2048}$

5. Find the area of one leaf of the rose

$r = 2\cos 3\theta.$



a. $\frac{\pi}{12}$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{3}$

d. $\frac{2\pi}{3}$

e. $\frac{4\pi}{3}$

6. Set up, but do not compute, the integral $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$ over the solid region between the paraboloid $z = 2x^2 + 2y^2$ and the plane $z = 8$ where $\vec{F} = (x^3, y^3, z(x^2+y^2))$.

a. $\int_0^{\pi} \int_0^2 \int_{2r^2}^8 4r^2\, dz\, dr\, d\theta$

b. $\int_0^{2\pi} \int_0^2 \int_{2r^2}^8 4r^2\, dz\, dr\, d\theta$

c. $\int_0^{2\pi} \int_0^2 \int_{2r^2}^8 4r^3\, dz\, dr\, d\theta$

d. $\int_0^{2\pi} \int_0^8 \int_0^{z/2} 4r^2\, dr\, dz\, d\theta$

e. $\int_0^{2\pi} \int_0^8 \int_0^{z/2} 4r^3\, dr\, dz\, d\theta$

7. Find the average value of $f = z$ on the solid hemisphere $0 \le z \le \sqrt{9 - x^2 - y^2}$.
Note: The average value of a function on a solid is $f_{\text{ave}} = \frac{1}{V} \iiint_V f\, dV$.

a. $\frac{3\pi}{8}$

b. $\frac{\pi}{2}$

c. $\frac{3}{2}$

d. $\frac{1}{2}$

e. $\frac{9}{8}$

8. Compute the line integral $\int \vec{\nabla} \times \vec{F} \cdot d\vec{s}$ counterclockwise around one loop of the helix $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta, 3\theta)$ for the vector field $\vec{F} = (xz, yz, z^2)$.
Hint: Compute $\vec{\nabla} \times \vec{F}$ in rectangular coordinates before integrating.

a. 16π

b. 32π

c. 64π

d. 128π

e. 0

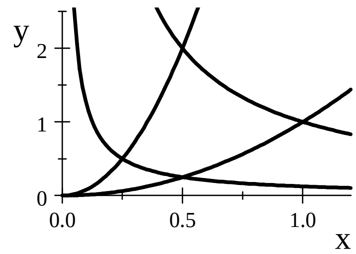
Work Out: (Points indicated. Part credit possible. Show all work.)

9. (20 points) A rectangular solid box is sitting on the xy-plane with its upper 4 vertices on the ellipsoid $x^2 + 4y^2 + 9z^2 = 108$. Find the dimensions and volume of the largest such box.
Full credit for solving by Lagrange multipliers.
Half credit for solving by Eliminating a Variable.
50% extra credit for solving both ways.

10. (20 points) Compute the integral $\iint y\,dA$ over the region in the first quadrant bounded by

$y = x^2$, $y = 8x^2$, $y = \dfrac{1}{x}$, and $y = \dfrac{1}{8x}$.

Use the following steps:



a. Define the curvilinear coordinates u and v by $y = u^3x^2$ and $y = \dfrac{v^3}{x}$.
Express the coordinate system as a position vector.

$\vec{r}(u,v) =$

b. Find the coordinate tangent vectors:

$\vec{e}_u =$

$\vec{e}_v =$

c. Compute the Jacobian factor:

$J = \left| \dfrac{\partial(x,y)}{\partial(u,v)} \right|$

d. Compute the integral:

$\iint y\,dA =$

11. (20 points) Compute the flux $\iint_H \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F} = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, \frac{z}{\sqrt{x^2+y^2}} \right)$ outward through the upper half of the sphere $x^2+y^2+z^2 = 9$. Use the following steps:

a. Parametrize the hemisphere:

$\vec{R}(\theta,\varphi) =$

b. Find the tangent vectors:

$\vec{e}_\theta =$

$\vec{e}_\varphi =$

c. Find the normal vector:

$\vec{N} =$

d. Fix the orientation of the normal (if necessary):

$\vec{N} =$

e. Evaluate the vector field on the cylinder:

$\vec{F}\left(\vec{R}(\theta,\varphi)\right) =$

f. Evaluate the dot product:

$\vec{F} \cdot \vec{N} =$

g. Calculate the flux:

$\iint_H \vec{F} \cdot d\vec{S} =$