Problems 1-3:  **Do 2 of the 3 problems, only.**

Identify the differential equation as one of the following types:

a. Separable Equation  
b. Equation with Homogeneous Coefficients  
c. Linear Equation  
d. Bernoulli Equation  
e. Exact Equation

Then solve the initial value problem.

1. (15 points) \[ \frac{dy}{dx} + \frac{y}{x} = 2x^2y^2 \text{ with } y(1) = \frac{1}{2} \]
2. (15 points) \((-e^{-x} + e^y) \, dx + (xe^y + e^y) \, dy = 0\) with \(y(1) = 0\)

3. (15 points) \(\frac{dy}{dx} = \frac{x^2}{y^2} + \frac{y}{x}\) with \(y(1) = 3\)
4. (5 points) Which of the following is the direction field of the differential equation: \( \frac{dy}{dx} = xy \). Circle the correct answer:

a. 

b. 

c. 

d. 

5. (5 points) On the following direction field, draw the solution curve with the initial condition \( y(1) = 2 \).

6. (10 points) Find the general solution of the differential equation \( x^2 \frac{d^2y}{dx^2} + 6x \frac{dy}{dx} + 6y = 0 \).
7. (15 points) Consider the initial value problem
\[ 2 \frac{d^2y}{dt^2} + 8 \frac{dy}{dt} + 26y = 0 \quad \text{with} \quad y(0) = 2 \quad \text{and} \quad \frac{dy}{dt}(0) = -1 \]

a. (8 pts) Find the general solution of the differential equation.

b. (7 pts) Find the solution satisfying the initial conditions.

8. (5 points) The solution of an initial value problem of the form
\[ \frac{d^2y}{dt^2} + b \frac{dy}{dt} + ky = 0 \quad \text{with} \quad y(0) = 2 \quad \text{and} \quad \frac{dy}{dt}(0) = 1 \]
is graphed below.

What can you say about the signs and relative sizes of the coefficients, $b$ and $k$?