Name $\qquad$ NetID $\qquad$

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At the top of your worksheet put:

## > restart;

Your name.
Your NetID.
MATH 308 Sec 511 Exam 1
Feb 24, 2009
$>$ with(plots): with(DEtools):
Number each problem and part.

1. (15 points) Consider the differential equation $\frac{d y}{d x}+\frac{y}{x}=2 x^{2} y^{2}$.
a. (5 pts) Find the general solution.
b. (3 pts) Find the specific solution satisfying the initial condition $y(1)=\frac{1}{2}$.
c. (4 pts) Plot the direction field of the differential equation for $0 \leq x \leq 2$. Adjust the vertical range to a reasonable value.
d. (3 pts) Add to the direction field the solutions satisfying each of the initial conditions

$$
y(1)=\frac{1}{4}, \quad y(1)=\frac{1}{2}, \quad y(1)=1, \quad y(1)=2
$$

2. (15 points) Consider the initial value problem $\frac{d y}{d x}=\frac{x^{2}}{y^{2}}-1$ with $y(1)=2$.
a. (5 pts) Use Euler's method to estimate $y(2)$ using 5 steps. Suppress the output of the do loop. Plot the 6 points connected by straight line seqments. What is the estimated $y(2)$ ?
b. (3 pts) Use Euler's method to estimate $y(2)$ using 100 steps. Suppress the output of the do loop. Plot the 101 points connected by straight line seqments. What is the estimated $y(2)$ ?
c. (5 pts) Find a formula for $\frac{d^{2} y}{d x^{2}}$ in terms of only $x$ and $y$. Compute $\left.\frac{d^{2} y}{d x^{2}}\right|_{x=1}$.
d. (2 pts) Are your estimates for $y(2)$ using Euler's method, larger or smaller than the correct value? Why? This requires no new computations. It requires a sentence response.
