```
> restart;
```
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MATH 308 Sec 511 Exam 1
Feb 24, 2009

```
> with(plots): with(DEtools):
```
#1a
```
> deq:=diff(y(x),x)+y(x)/x=2*x^2*y(x)^2;
```

$$deq := \frac{d}{dx} y(x) + \frac{y(x)}{x} = 2\, x^2\, y(x)^2 \tag{1}$$

```
> dsolve(deq,y(x));
```

$$y(x) = \frac{1}{\left(-x^2 + \_C1\right) x} \tag{2}$$
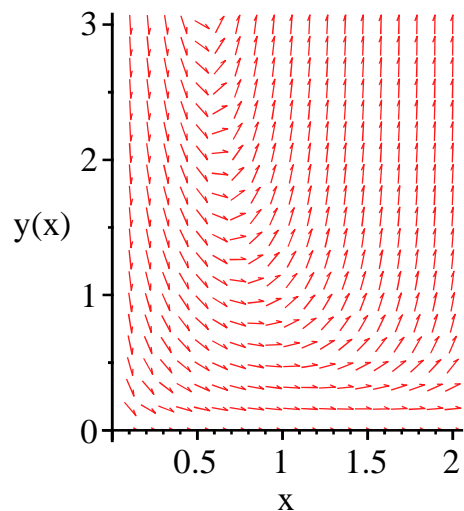
#1b
```
> init:=y(1)=1/2;
```

$$init := y(1) = \frac{1}{2} \tag{3}$$

```
> dsolve([deq,init],y(x));
```

$$y(x) = -\frac{1}{x\left(x^2 - 3\right)} \tag{4}$$
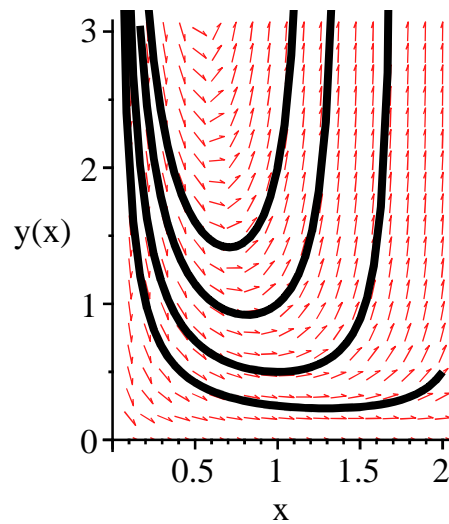
#1c
```
> DEplot(deq, y(x), x=0..2, y=0..3);
```


#1d
```
> DEplot(deq, y(x), x=0..2, y=0..3, [y(1)=1/4,y(1)=1/2,y(1)=1,y(1)
  =2], linecolor=black);
```

#2a

```
> deq:=diff(y(x),x)=x^2/y(x)^2-1;
```

$$deq := \frac{d}{dx}\,y(x) = \frac{x^2}{y(x)^2} - 1 \tag{5}$$

```
> X[0]:=1; Y[0]:=2;
```

$$X_0 := 1$$
$$Y_0 := 2 \tag{6}$$

```
> h:=.2;
```

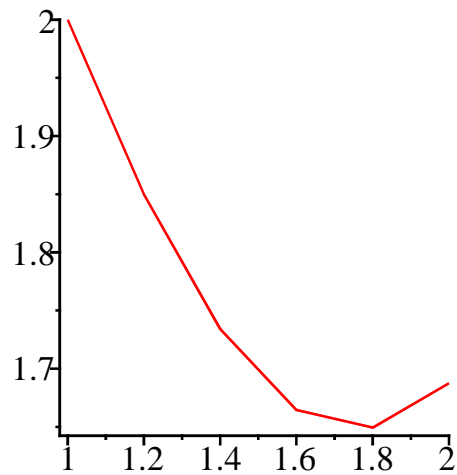$$h := 0.2 \tag{7}$$

```
> for i from 1 to 5 do
    m:=eval(rhs(deq), [x=X[i-1], y=Y[i-1]]);
    tanline:=y=Y[i-1]+m*(x-X[i-1]);
    X[i]:=X[i-1]+h; Y[i]:=eval(rhs(tanline),x=X[i]);
    end do:
> pts:=seq( [X[i],Y[i]], i=0..5 );
```

$pts := [1, 2], [1.2, 1.850000000], [1.4, 1.734149014], [1.6, 1.664499676], [1.8,$ $\tag{8}$
$\quad 1.649299916], [2.0, 1.687518551]$

```
> ppts10:=plot([ pts ]);%;
```

$$ppts10 := PLOT(\ldots)$$

y(2) is

```
> Y[5];
```

$$1.687518551 \tag{9}$$

#2b

```
> h:=.01;
```

$$h := 0.01 \tag{10}$$

```
> for i from 1 to 100 do
    m:=eval(rhs(deq), [x=X[i-1], y=Y[i-1]]);
    tanline:=y=Y[i-1]+m*(x-X[i-1]);
    X[i]:=X[i-1]+h; Y[i]:=eval(rhs(tanline),x=X[i]);
  end do:
```
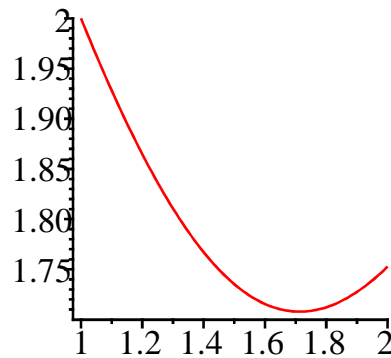
```
> pts:=seq( [X[i],Y[i]], i=0..100 );
```

$pts := [1, 2], [1.01, 1.992500000], [1.02, 1.985069485], [1.03, 1.977709759], [1.04, \tag{11}$
$1.970422131], [1.05, 1.963207920], [1.06, 1.956068446], [1.07, 1.949005038], [1.08,$
$1.942019027], [1.09, 1.935111747], [1.10, 1.928284534], [1.11, 1.921538726], [1.12,$
$1.914875660], [1.13, 1.908296674], [1.14, 1.901803104], [1.15, 1.895396281], [1.16,$
$1.889077534], [1.17, 1.882848186], [1.18, 1.876709553], [1.19, 1.870662945], [1.20,$
$1.864709662], [1.21, 1.858850994], [1.22, 1.853088220], [1.23, 1.847422605], [1.24,$
$1.841855400], [1.25, 1.836387842], [1.26, 1.831021150], [1.27, 1.825756525], [1.28,$
$1.820595147], [1.29, 1.815538176], [1.30, 1.810586749], [1.31, 1.805741979], [1.32,$
$1.801004953], [1.33, 1.796376730], [1.34, 1.791858343], [1.35, 1.787450794], [1.36,$
$1.783155054], [1.37, 1.778972061], [1.38, 1.774902719], [1.39, 1.770947896], [1.40,$
$1.767108425], [1.41, 1.763385100], [1.42, 1.759778677], [1.43, 1.756289871], [1.44,$
$1.752919355], [1.45, 1.749667760], [1.46, 1.746535674], [1.47, 1.743523640], [1.48,$
$1.740632157], [1.49, 1.737861676], [1.50, 1.735212603], [1.51, 1.732685296], [1.52,$
$1.730280064], [1.53, 1.727997168], [1.54, 1.725836820], [1.55, 1.723799184], [1.56,$
$1.721884370], [1.57, 1.720092444], [1.58, 1.718423417], [1.59, 1.716877253], [1.60,$
$1.715453865], [1.61, 1.714153116], [1.62, 1.712974821], [1.63, 1.711918744], [1.64,$
$1.710984602], [1.65, 1.710172063], [1.66, 1.709480746], [1.67, 1.708910226], [1.68,$

1.708460029], [1.69, 1.708129638], [1.70, 1.707918490], [1.71, 1.707825978], [1.72, 1.707851454], [1.73, 1.707994227], [1.74, 1.708253566], [1.75, 1.708628702], [1.76, 1.709118828], [1.77, 1.709723098], [1.78, 1.710440635], [1.79, 1.711270523], [1.80, 1.712211818], [1.81, 1.713263542], [1.82, 1.714424688], [1.83, 1.715694222], [1.84, 1.717071082], [1.85, 1.718554180], [1.86, 1.720142407], [1.87, 1.721834629], [1.88, 1.723629694], [1.89, 1.725526428], [1.90, 1.727523642], [1.91, 1.729620127], [1.92, 1.731814662], [1.93, 1.734106014], [1.94, 1.736492934], [1.95, 1.738974165], [1.96, 1.741548440], [1.97, 1.744214485], [1.98, 1.746971019], [1.99, 1.749816754], [2.00, 1.752750401]

> **ppts10:=plot([pts]);%;**

$$ppts10 := PLOT(\ldots)$$

y(2) is

> **Y[100];**

$$1.752750401 \tag{12}$$

#2c

> **deq2:=diff(deq,x);**

$$deq2 := \frac{d^2}{dx^2}\, y(x) = \frac{2\,x}{y(x)^2} - \frac{2\,x^2\left(\dfrac{d}{dx}\,y(x)\right)}{y(x)^3} \tag{13}$$

> **deq2rhs:=simplify(eval(rhs(deq2),deq));**

$$deq2rhs := \frac{2\,x\left(y(x)^3 - x^3 + x\,y(x)^2\right)}{y(x)^5} \tag{14}$$

> **eval(deq2rhs,[x=1,y(x)=2]);**

$$\frac{11}{16} \tag{15}$$

#2d

They are smaller than the correct values because the solution is concave up.