Name_____ ID_____

MATH 308 Exam I Fall 2000

Section 200 P. Yasskin

1	/10	5	/10
2	/10	6	/15
3	/10	7	/20
4	/10	8	/15

HAND COMPUTATIONS

Problems 1-5: Solve the initial value problem using the method appropriate to one of the following types of first order differential equations:

- **a**. Separable Equation
- **b**. Equation with Homogeneous Coefficients
- **c**. Linear Equation
- **d**. Bernoulli Equation Hint: $k = \frac{1}{1-s}$
- **e**. Exact Equation

Be sure to identify the equation type. There is one problem for each equation type.

1. (10 points) $\frac{dy}{dx} = \frac{4y}{x} + x$ with $y(1) = 0$

2. (10 points) $\frac{dy}{dt} = -\frac{2y}{t} + 6t\sqrt{y}$ with $y(1) = 9$

3. (10 points) $\frac{dx}{dt} = \frac{x}{t} + \frac{1}{tx^2}$ with $x(1) = 2$

4. (10 points) $\frac{dy}{dx} = \frac{y\sin x - \sin y}{x\cos y + \cos x}$ with $y(0) = 2$

5. (10 points) $\frac{dy}{dx} = \frac{x^2}{y^2} + \frac{y}{x}$ with $y(1) = 3$

6. (15 points) Set up the differential equation and initial condition for $P(t)$ in the following problem. Do not solve the equations.

A certain pond can contain $12,000$ ft^3 of water before the dam overflows. Initially, there are $8,000$ ft^3 of water and 50 gallons of pollution in the pond. Acme Polluters is putting 2 gallons of pollution in the pond a day. Every day, $2,000$ ft^3 of fresh water is pumped into the pond and $1,000$ ft^3 of polluted water is pumped out. Let $P(t)$ be the gallons of pollution in the pond after t days. When does the dam overflow and what is the concentration of the polution in the water when the dam first overflows?

MAPLE COMPUTATIONS

7. (20 points) Consider the differential equation $\frac{dy}{dt} = e^t - y$.

- **a**. (4 pts) Find the general solution.
- **b**. (4 pts) Find the specific solution satisfying the initial condition $y(0) = 4$.
- **c**. (4 pts) Plot the direction field of the differential equation for times $-4 \le t \le 6$.
- **d**. (4 pts) Add to the direction field the solutions satisfying each of the initial conditions

  $$y(0) = -4, \quad y(0) = -2, \quad y(0) = 0, \quad y(0) = 2, \quad y(0) = 4, \quad y(0) = 6$$

  Adjust the vertical range to a reasonable value.
- **e**. (4 pts) Looking at the exact solution and the plot, describe the behavior of the solutions for large times. At about what time does this asymptotic behavior begin?

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8. (15 points) Consider the initial value problem $\frac{dy}{dx} = \cos x + \sin y$ with $y(0) = 1$.

- **a**. (10 pts) What happens if you try to solve the equations using **dsolve** ? Find a Taylor polynomial approximation about $x = 0$ for the solution to this initial value problem keeping terms up to and including x^4.
- **b**. (5 pts) Plot the direction field for this differential equation together with the solution satisfying the initial condition. Plot the Taylor polynomial approximation. Combine the two plots into a single plot. On approximately what interval is the Taylor polynomial a good approximation?

To Turn in Your Maple Computations:

1. Save your Maple file as lastname_exam1.mws
2. Print your file as follows:
   a. Click on **FILE**, **PRINT** and **Printer Command**.
   b. Make the command read: **lpr -J "Yasskin Maple Exam 1"**
   c. Call Dr. Yasskin over to check your printing.
   d. Click on **PRINT**.
3. Mail your file as follows:
   a. Start the mail program: **pine**
   b. Compose a letter by typing **C**.
   c. In the header region, enter:
      To **yasskin**
      Attachment **lastname_exam1.mws** (or the *exact* name of your Maple file)
      Subject **Last Name Exam1**
   d. Call Dr. Yasskin over to check your email.
   e. Mail the letter by typing **^X** and **Y**.