Name________________________ ID____________

1	/10	6	/10
2	/10	7	/15
5	/10	8	/15
Scantron			/30

MATH 311 Exam 1 Fall 2000

Section 502 P. Yasskin

1. (10 points) Find the inverse of $A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 1 & 4 \\ 1 & 1 & 0 \end{pmatrix}$.

Use it to solve $XA = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$.

2. (10 points) Consider the polynomials

$$p_1(x) = 1 - x^2$$
$$p_2(x) = 2 - x - x^2$$
$$p_3(x) = 1 - x$$

and the vector space

$$W = \text{Span}(p_1, p_2, p_3).$$

Find a subset of $\{p_1, p_2, p_3\}$ which is a basis for W. Prove it spans W and is linearly independent.

3. Consider the vector space P_3, the set of polynomials of degree 3 or less?

- (5 points) Scantron #1 Which of the following is NOT a subspace of P_3?
  - **a.** $A = \{ \; p \in P_3 \; | \; p(0) = 0 \; \}$
  - **b.** $B = \{ \; p \in P_3 \; | \; p(1) = 0 \; \}$
  - **c.** $C = \{ \; p \in P_3 \; | \; p(0) = p(1) \; \}$
  - **d.** $D = \{ \; p \in P_3 \; | \; p(0) + p(1) = 0 \; \}$
  - **e.** $E = \{ \; p \in P_3 \; | \; p(0) = 1 \; \}$

4. Consider the vector space $\mathbf{R}^+$ of all positive real numbers with the operations of

Vector Addition: $x \oplus y = xy$ (real number addition)

Scalar Multiplication: $\alpha \circ x = x^\alpha$ (real number exponentiation)

- (5 points) Scantron #2 Translate the vector identity

$$0 \circ x = \vec{0}$$

into ordinary arithmetic.
  - **a.** $1^x = 1$
  - **b.** $x^0 = 1$
  - **c.** $0^x = 0$
  - **d.** $x^1 = x$
  - **e.** $0^x = 1$

5. Consider the linear map $L : \mathbf{R}^3 \to \mathbf{R}^4$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}$.

- (10 points) Solve $L(\vec{x}) = \begin{pmatrix} 2 \\ -1 \\ 2 \\ 4 \end{pmatrix}$.

- (5 points) Scantron #3 Describe the solution set:
  - **a.** No Solutions
  - **b.** Unique Solution (Point in $\mathbf{R}^3$)
  - **c.** ∞-Many Solutions (Line in $\mathbf{R}^3$)
  - **d.** ∞-Many Solutions (Plane in $\mathbf{R}^3$)
  - **e.** ∞-Many Solutions (All of $\mathbf{R}^3$)

- (5 points) Scantron #4 Is L a one-to-one function?
  - **a.** Yes
  - **b.** No

6. Again consider the linear map $L : \mathbf{R}^3 \to \mathbf{R}^4$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}$.

- (10 points) Solve $L(\vec{x}) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

- (5 points) Scantron #5 Describe the solution set:
  a. No Solutions
  b. Unique Solution (Point in $\mathbf{R}^3$)
  c. ∞-Many Solutions (Line in $\mathbf{R}^3$)
  d. ∞-Many Solutions (Plane in $\mathbf{R}^3$)
  e. ∞-Many Solutions (All of $\mathbf{R}^3$)

- (5 points) Scantron #6 Is L an onto function?
  a. Yes
  b. No

7. Again consider the linear map $L : \mathbf{R}^3 \to \mathbf{R}^4$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}$.

- (5 points) Find $Ker(L)$, the kernel (or null space) of L.

- (5 points) Give a basis for $Ker(L)$. (No proof)

- (5 points) What is the dimension of $Ker(L)$? (No proof)

8. Again consider the linear map $L : \mathbf{R}^3 \to \mathbf{R}^4$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}$.

- (5 points) Find $Im(L)$, the image (or range) of L.

- (5 points) Give a basis for $Im(L)$. (No proof)

- (5 points) What is the dimension of $Im(L)$? (No proof)