Name________________________ ID____________

1	/15	3	/15
2	/30	4	/45

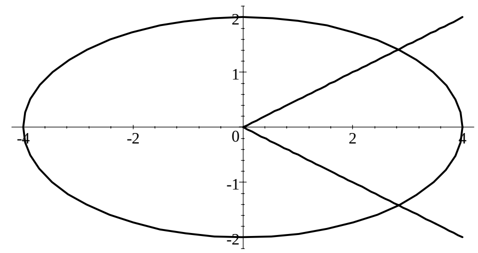
MATH 311 Exam 3 Fall 2000

Section 502 P. Yasskin

1. (15 points) Compute $\iint_R x\,dx\,dy$ over the region inside the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$ between the lines $y = \dfrac{x}{2}$ and $y = -\dfrac{x}{2}$ in the 1^{st} and 4^{th} quadrants.

HINT: Use the elliptic coordinate system

$$x = 4t\cos\theta, \quad y = 2t\sin\theta.$$

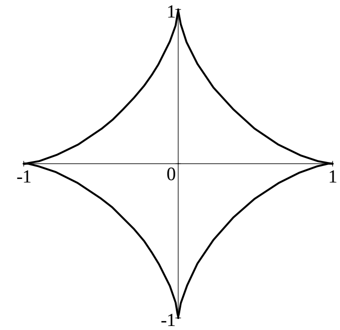


2. (30 points) The diamond shaped region at the right is called an astroid. It is the graph of

$$x^{2/3} + y^{2/3} = 1$$

and it may be parametrized by

$$x = \cos^3 t \qquad y = \sin^3 t.$$



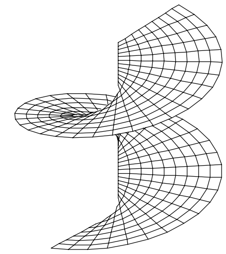
a. Find its arc length. HINT: First find one quarter of the length.

b. Find its area. HINT: What does Green's Theorem say about the integral $\oint(-y\,dx + x\,dy)$?

3. (15 points) Compute $\int \vec{F} \bullet d\vec{s}$ for $\vec{F} = (z^2 - (\sin x)e^{\cos x}, (\cos y)e^{\sin y}, 2xz)$ along the helix $\vec{r}(\theta) = (\cos\theta, \sin\theta, \theta)$ for $0 \le \theta \le 2\pi$.

HINT: Check $\vec{F}$ is a conservative vector field. Then find a potential OR change the path.

4. (45 points) The spiral ramp at the right may be parametrized as



$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$$

for $0 \le r \le 2$ and $0 \le \theta \le 3\pi$.

a. Find the tangent plane at the point $(x,y,z) = (-1,0,\pi)$. Give its equation in both parametric form and non-parametric form.

b. Compute $\iint \sqrt{x^2+y^2}\ dS$ over this spiral ramp.

c. Compute $\iint \vec{F} \bullet d\vec{S}$ over this spiral ramp if $\vec{F} = (xz, yz, z^2)$.