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Math 311 Exam 1 Spring 2002

Section 503 P. Yasskin

1	/10
2	/10
3	/30
4	/25
5	/25

1. (10 points) A matrix A satisfies $E_3 E_2 E_1 A = U$ where

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 5 & * \\ 0 & -3 & * \\ 0 & 0 & -1 \end{pmatrix}$$

and the $*$'s represent unknown non-zero numbers. Find $\det A$.

2. (10 points) If c is a scalar, A is a 50×60 matrix and B is a 60×80 matrix, prove $A(cB) = c(AB)$.
HINT: Write out the ij-component of each side.

3. (30 points) Consider the triangle with vertices

$$A = (2,4,0) \qquad B = (4,2,1) \qquad C = (2,7,4)$$

a. Find $\cos\theta$ where θ is the angle at vertex A.

b. Find the area of the triangle ΔABC.

#3 CONTINUED $A = (2, 4, 0)$ $B = (4, 2, 1)$ $C = (2, 7, 4)$

c. Find a set of parametric equations for the line containing A and C.

d. Find a set of parametric equations for the plane containing A, B and C.

e. Find a non-parametric equation for the plane containing A, B and C.

4. (25 points) Consider the system of equations:

$$AX = B \qquad \text{where} \qquad A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \qquad X = \begin{pmatrix} x & p \\ y & q \\ z & r \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}$$

Compute A^{-1}. (Give reasons for each step.)

Solve $AX = B$.

5. (25 points) Consider the system of equations:

$$\begin{aligned} 3w + 6x + y \phantom{{}-3z} &= 5 \\ y - 3z &= 2 \\ w + 2x + y - 2z &= 3 \\ -2w - 4x + y - 5z &= b \end{aligned}$$

Find the value(s) of b for which there exist solutions. (Give reasons for each step.)

For that value (those values) of b what is the solution set?

Give a geometrical description of the solution set.