1. (10 points) A matrix $A$ satisfies $E_3E_2E_1A = U$ where

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 2 & 5 & * \\ 0 & -3 & * \\ 0 & 0 & -1 \end{pmatrix}$$

and the *’s represent unknown non-zero numbers. Find $\det A$.

2. (10 points) If $c$ is a scalar, $A$ is a $50 \times 60$ matrix and $B$ is a $60 \times 80$ matrix, prove $A(cB) = c(AB)$.

HINT: Write out the $ij$-component of each side.
3. (30 points) Consider the triangle with vertices

\[ A = (2, 4, 0) \quad B = (4, 2, 1) \quad C = (2, 7, 4) \]

a. Find \( \cos \theta \) where \( \theta \) is the angle at vertex \( A \).

b. Find the area of the triangle \( \triangle ABC \).
c. Find a set of parametric equations for the line containing $A$ and $C$.

d. Find a set of parametric equations for the plane containing $A$, $B$ and $C$.

e. Find a non-parametric equation for the plane containing $A$, $B$ and $C$. 

$A = (2, 4, 0) \quad B = (4, 2, 1) \quad C = (2, 7, 4)$
4. (25 points) Consider the system of equations:

\[ AX = B \quad \text{where} \quad A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{pmatrix} \]

Compute \( A^{-1} \). (Give reasons for each step.)

Solve \( AX = B \).
5. (25 points) Consider the system of equations:

\begin{align*}
3w + 6x + y &= 5 \\
y - 3z &= 2 \\
w + 2x + y - 2z &= 3 \\
-2w - 4x + y - 5z &= b
\end{align*}

Find the value(s) of \( b \) for which there exist solutions. (Give reasons for each step.)

For that value (those values) of \( b \) what is the solution set?

Give a geometrical description of the solution set.