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Math 311 Final Exam Spring 2002

Section 503 P. Yasskin

1 EC	/10	3	/30
3	/40	4	/30

1. (10 points Extra Credit) Determine if and where the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$

intersects the plane $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \\ 3 \end{pmatrix} + r \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$.

2. (40 points) Let $M(p,q)$ be the vector space of $p \times q$ matrices, and let $P = \begin{pmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{pmatrix}$.

Consider the linear function

$$L : M(2,2) \to M(3,2) \quad \text{given by} \quad L(X) = PX$$

a. (5) Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and compute $L(X)$.

b. (2) Identify the domain of L. What is its dimension?

c. (2) Identify the codomain of L. What is its dimension?

d. (8) Identify the kernel (null space) of L. Give a basis and the dimension.

Problem **2** continued:

e. (8) Identify the image (range) of L. Give a basis and the dimension.

f. (3) Is L onto? Why?

g. (3) Is L one-to-one? Why?

h. (2) Check that the dimensions of the kernel and image are consistent with the dimensions of the domain and codomain.

Problem **2** continued:

i. (7) Find the matrix of L relative to the bases

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$F_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad F_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \quad F_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$F_4 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad F_5 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad F_6 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

3. (30 points) On the vector space $P_2 = \{\text{polynomials of degree less than } 2\}$ consider the function of two polynomials given by

$$\langle p, q \rangle = \int_0^\infty p(x)q(x)e^{-x}\,dx$$

a. (15) Show $\langle p, q \rangle$ is an inner product.

b. (15) Find the angle θ between the polynomials $p(x) = 1$ and $q(x) = x$.

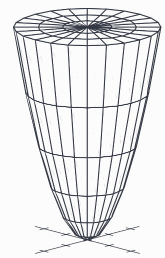
You may use these integrals without proof:

$$\int_0^\infty e^{-x}\,dx = 1, \quad \int_0^\infty xe^{-x}\,dx = 1, \quad \int_0^\infty x^2e^{-x}\,dx = 2, \quad \int_0^\infty x^3e^{-x}\,dx = 6$$

4. (30 points) Gauss' Theorem states that if V is a volume in $\mathbf{R}^3$ and ∂V is its boundary surface oriented outward from V and $\vec{F}$ is a nice vector field on V then

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot \overrightarrow{dS}$$

Verify Gauss' Theorem if $\vec{F} = (xz^2, -yz^2, x^2 z + y^2 z)$ and V is the volume above the paraboloid $z = x^2 + y^2$ and below the plane $z = 9$. Notice that ∂V consists of the paraboloid P and a disk D. Be sure to use the correct orientations for P and D.



a. (6) Compute $\displaystyle\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$:

i. $\vec{\nabla} \cdot \vec{F} =$

ii. $\displaystyle\iiint_V \vec{\nabla} \cdot \vec{F}\, dV =$

Problem **4** continued: Recall $\vec{F} = (xz^2, -yz^2, x^2z + y^2z)$.

b. (10) For the paraboloid P compute $\iint_P \vec{F} \cdot \overrightarrow{dS}$:

i. $\vec{R}(r,\theta) =$

ii. $\vec{R}_r =$

iii. $\vec{R}_\theta =$

iv. $\vec{N} =$

v. $\vec{F}\left(\vec{R}(r,\theta)\right) =$

vi. $\vec{F}\left(\vec{R}(r,\theta)\right) \cdot \vec{N} =$

vii. $\iint_P \vec{F} \cdot \overrightarrow{dS} =$

Problem **4** continued: Recall $\vec{F} = (xz^2, -yz^2, x^2z + y^2z)$.

c. (10) For the disk D compute $\iint_D \vec{F} \cdot \overrightarrow{dS}$:

i. $\vec{R}(r,\theta) =$

ii. $\vec{R}_r =$

iii. $\vec{R}_\theta =$

iv. $\vec{N} =$

v. $\vec{F}\left(\vec{R}(r,\theta)\right) =$

vi. $\vec{F}\left(\vec{R}(r,\theta)\right) \cdot \vec{N} =$

vii. $\iint_D \vec{F} \cdot \overrightarrow{dS} =$

d. (4) Verify the two sides of Gauss' Theorem are equal.