

Name \_\_\_\_\_

Math 311                  Exam 1                  Spring 2010

Section 502                                  P. Yasskin

1	/25	4	/10
2	/25	5	/20
3	/25	Total	/105

1. (25 points) Consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 3 \end{pmatrix} \quad X = \begin{pmatrix} a & p \\ b & q \\ c & r \\ d & s \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$$

a. Compute  $A^{-1}$ .

b. Solve the equation  $AX = B$ .

2. (25 points) Consider the system of equations:

$$\begin{aligned} 3x + 5y - 2z &= 17 \\ x + y &= 5 \\ 3y - 3z &= p \end{aligned}$$

a. Write out the augmented matrix and row reduce it to reduced row echelon form. (Give reasons for each step.)

b. For what value(s) of  $p$  are there (At least one answer is "No  $p$ ".)

no solutions?

a unique solution?

exactly two solutions?

infinitely-many solutions?

c. For those  $p$ 's for which there are solutions, what are the solutions?

3. (25 points) Consider the system of equations:

$$\begin{aligned} 3x + 5y - 2z &= 17 \\ x + y &= 5 \\ 2y - z &= p \end{aligned}$$

a. Write out the augmented matrix and row reduce it to reduced row echelon form. (Give reasons for each step.)

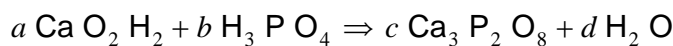
b. For what value(s) of  $p$  are there (At least one answer is "No  $p$ ".)

no solutions?  a unique solution?

exactly two solutions?  infinitely-many solutions?

c. For those  $p$ 's for which there are solutions, what are the solutions?

4. (10 points) Write out the equations which need to be solved to find  $a, b, c$  and  $d$  to balance the chemical equation:



Then set up the augmented matrix for the equations. Do not solve the equations.

5. (20 points) Say whether each of the following statements is true or false for all scalars  $a, b$  and  $c$ , all  $3 \times 3$  matrices  $A, B$  and  $C$  and all  $3 \times 3$  elementary matrices  $E_1, E_2$  and  $E_3$  of types I, II and III respectively where  $E_1, E_2$  and  $E_3$  are NOT the unit matrix. Circle your answers.

- |    |   |      |       |
|----|---|------|-------|
| a. | $(A + B)C = AC + BC$ .....                              | True | False |
| b. | $(A + B)^2 = A^2 + 2AB + B^2$ .....                     | True | False |
| c. | $(AB)^2 = A^2B^2$ .....                                 | True | False |
| d. | If $B = A - A^T$ then $B^T = -B$ .....                  | True | False |
| e. | $(AB)^T = A^T B^T$ .....                                | True | False |
| f. | $(AB)^T = B^T A^T$ .....                                | True | False |
| g. | $(AB)^{-1} = A^{-1} B^{-1}$ .....                       | True | False |
| h. | $(AB)^{-1} = B^{-1} A^{-1}$ .....                       | True | False |
| i. | $(aA + bB)^T = (aA^T + bB^T)$ .....                     | True | False |
| j. | $(aA + bB)^{-1} = (aA^{-1} + bB^{-1})$ .....            | True | False |
| k. | $E_1 A = A E_1^T$ .....                                 | True | False |
| l. | $E_2 A = A^T E_2^T$ .....                               | True | False |
| m. | $E_3 A = (A^T E_3^T)^T$ .....                           | True | False |
| n. | $\det(cA) = c^3 \det A$ .....                           | True | False |
| o. | $\det(AB^{-1}) = \det A - \det B$ .....                 | True | False |
| p. | $\det(AB^T) = \det A \det B$ .....                      | True | False |
| q. | $\det(E_1 A) = \frac{1}{\det A}$ .....                  | True | False |
| r. | $\det(E_2 A) = k \det A$ with $k \neq 1$ .....          | True | False |
| s. | $\det(E_3 A) = k \det A$ with $k \neq 1$ .....          | True | False |
| t. | $\det(E_1 E_2 E_3 A E_3^{-1} E_2^{-1}) = -\det A$ ..... | True | False |