Name_____

Math 311 Exam 1 Spring 2010

Section 502 P. Yasskin

1	/25	4	/10
2	/25	5	/20
3	/25	Total	/105

1. (25 points) Consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 3 \end{pmatrix} \qquad X = \begin{pmatrix} a & p \\ b & q \\ c & r \\ d & s \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$$

a. Compute A^{-1}.

b. Solve the equation $AX = B$.

2. (25 points) Consider the system of equations:

$$\begin{aligned} 3x + 5y - 2z &= 17 \\ x + y &= 5 \\ 3y - 3z &= p \end{aligned}$$

a. Write out the augmented matrix and row reduce it to reduced row eschelon form. (Give reasons for each step.)

b. For what value(s) of p are there (At least one answer is "No p".)

no solutions? [ ] a unique solution? [ ]

exactly two solutions? [ ] infinitely-many solutions? [ ]

c. For those p's for which there are solutions, what are the solutions?

3. (25 points) Consider the system of equations:

$$\begin{aligned} 3x + 5y - 2z &= 17 \\ x + y &= 5 \\ 2y - z &= p \end{aligned}$$

a. Write out the augmented matrix and row reduce it to reduced row eschelon form. (Give reasons for each step.)

b. For what value(s) of p are there (At least one answer is "No p".)

no solutions? [ ] a unique solution? [ ]

exactly two solutions? [ ] infinitely-many solutions? [ ]

c. For those p's for which there are solutions, what are the solutions?

4. (10 points) Write out the equations which need to be solved to find a, b, c and d to balance the chemical equation:

$$a\ \mathrm{Ca\ O_2\ H_2} + b\ \mathrm{H_3\ P\ O_4} \Rightarrow c\ \mathrm{Ca_3\ P_2\ O_8} + d\ \mathrm{H_2\ O}$$

Then set up the augmented matrix for the equations. Do not solve the equations.

5. (20 points) Say whether each of the following statements is true or false for all scalars a, b and c, all 3×3 matrices A, B and C and all 3×3 elementary matrices E_1, E_2 and E_3 of types I, II and III respectively where E_1, E_2 and E_3 are NOT the unit matrix. Circle your answers.

a. $(A+B)C = AC + BC$ ..... True False
b. $(A+B)^2 = A^2 + 2AB + B^2$ ..... True False
c. $(AB)^2 = A^2B^2$ ..... True False
d. If $B = A - A^{\mathsf{T}}$ then $B^{\mathsf{T}} = -B$ ..... True False
e. $(AB)^{\mathsf{T}} = A^{\mathsf{T}}B^{\mathsf{T}}$ ..... True False
f. $(AB)^{\mathsf{T}} = B^{\mathsf{T}}A^{\mathsf{T}}$ ..... True False
g. $(AB)^{-1} = A^{-1}B^{-1}$ ..... True False
h. $(AB)^{-1} = B^{-1}A^{-1}$ ..... True False
i. $(aA+bB)^{\mathsf{T}} = \left(aA^{\mathsf{T}} + bB^{\mathsf{T}}\right)$ ..... True False
j. $(aA+bB)^{-1} = (aA^{-1} + bB^{-1})$ ..... True False
k. $E_1A = AE_1^{\mathsf{T}}$ ..... True False
l. $E_2A = A^{\mathsf{T}}E_2^{\mathsf{T}}$ ..... True False
m. $E_3A = \left(A^{\mathsf{T}}E_3^{\mathsf{T}}\right)^{\mathsf{T}}$ ..... True False
n. $\det(cA) = c^3 \det A$ ..... True False
o. $\det(AB^{-1}) = \det A - \det B$ ..... True False
p. $\det\left(AB^{\mathsf{T}}\right) = \det A \det B$ ..... True False
q. $\det(E_1A) = \dfrac{1}{\det A}$ ..... True False
r. $\det(E_2A) = k \det A$ with $k \neq 1$ ..... True False
s. $\det(E_3A) = k \det A$ with $k \neq 1$ ..... True False
t. $\det(E_1E_2E_3AE_3^{-1}E_2^{-1}) = -\det A$ ..... True False