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Math 311 Exam 2 Spring 2010

Section 502 P. Yasskin

1	/45	3	/30
2	/40	Total	/115

1. (45 points) Consider the vector space of functions $V = Span(1, \cos x, \cos^2 x)$ with basis

$$v_1 = 1 \qquad v_2 = \cos x \qquad v_3 = \cos^2 x$$

a. (5 pts) Show $\cos 2x \in V$.
HINT: Use a trig identity.

b. (10 pts) Show another basis for V is

$$p_1 = 1 \qquad p_2 = \cos x \qquad p_3 = \cos 2x$$

c. (6 pts) Find the change of basis matrix from the p basis to the v basis. Call it $\underset{v \leftarrow p}{C}$.

d. (6 pts) Find the change of basis matrix from the v basis to the p basis. Call it $\underset{p \leftarrow v}{C}$.

e. (6 pts) Consider the linear function $L : V \longrightarrow \mathbb{R}^2$ given by $L(f) = \begin{pmatrix} f(0) \\ f\left(\frac{\pi}{2}\right) \end{pmatrix}$.

Find the matrix of L relative to the v basis for V and the basis $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for $\mathbb{R}^2$. Call it $\underset{e \leftarrow v}{A}$.

f. (6 pts) Find the matrix of L relative to the p basis for V and the e basis for $\mathbb{R}^2$. Call it $\underset{e \leftarrow p}{B}$.

g. (6 pts) Recompute $\underset{e \leftarrow p}{B}$ by a second method.
(One method should use the change of basis matrix, the other should not.)

2. (40 points) Consider a linear map $L : \mathbf{R}^n \to \mathbf{R}^p$ whose matrix relative to the standard bases is

$$A = \begin{pmatrix} 1 & -2 & 0 & -4 \\ -1 & 2 & 1 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

a. (4 pts) What are n and p?

b. (9 pts) Identify the kernel of L, a basis for the kernel, and the dimension of the kernel.

c. (9 pts) Identify the image of L, a basis for the image, and the dimension of the image.

d. (6 pts) Is the function L one-to-one? Why?

e. (6 pts) Is the function L onto? Why?

f. (6 pts) Verify the dimensions in a, b and c agree with the Nullity-Rank Theorem.

3. (30 points) Consider the vector space of functions $V = Span(1, \cos x, \cos^2 x)$ with the inner product $\langle f, g \rangle = \int_0^\pi f(x)g(x)\sin x\, dx$.

Here is a table of integrals:

$$\int_0^\pi \sin x\, dx = 2 \qquad \int_0^\pi \cos x \sin x\, dx = 0 \qquad \int_0^\pi \cos^2 x \sin x\, dx = \frac{2}{3}$$

$$\int_0^\pi \cos^3 x \sin x\, dx = 0 \qquad \int_0^\pi \cos^4 x \sin x\, dx = \frac{2}{5} \qquad \int_0^\pi \cos^5 x \sin x\, dx = 0$$

a. (10 pts) Find the angle between the functions $p = 1$ and $q = \cos^2 x$.

b. (20 pts) Apply the Gram-Schmidt procedure to the basis

$$v_1 = 1 \qquad v_2 = \cos x \qquad v_3 = \cos^2 x$$

to produce an orthogonal basis w_1, w_2, w_3 and an orthonormal basis u_1, u_2, u_3.