Name________________________________

Math 311 Exam 3 Spring 2010

Section 502 P. Yasskin

1,2	/ 4	10	/15
3,4	/12	11,12	/12
5	/10	13	/15
6,7	/12	14,15	/10
8,9	/16	Total	/106

Consider the vector space P_3 of polynomials with degree less than 3 with standard basis

$$e_1 = 1 \qquad e_2 = x \qquad e_3 = x^2$$

and the linear operator

$$L : P_3 \to P_3 : L(p) = (x-2)\frac{dp}{dx}$$

1. (2 pts) What is $\dim P_3$? What is the size of the matrix A of the linear map L?

$\dim P_3 =$ ☐ A is a ☐ matrix.

2. (2 pts) Let $p = a + bx + cx^2$. Compute $L(p)$.

$L(p) =$

3. (10 pts) Identify the kernel of L, a basis for the kernel, and the dimension of the kernel.

$\dim Ker(L) =$ ☐

4. (2 pts) What does the kernel of L, (found in part 2), say about one of the eigenvalues of L, and the eigenpolynomial(s) for that eigenvalue? No new computations!

5. (10 pts) Identify the image of L, a basis for the image, and the dimension of the image.

$\dim Im(L) =$ ☐

6. (6 pts) Is the function L one-to-one? Why?

7. (6 pts) Is the function L onto? Why?

8. (6 pts) Find the matrix of the linear map L relative to the e basis. Call it $\underset{e \leftarrow e}{A}$.

$$\underset{e \leftarrow e}{A} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

9. (10 pts) Find the eigenvalues of $\underset{e \leftarrow e}{A}$.

$\lambda_1 = \square \quad \lambda_2 = \square \quad \lambda_3 = \square$

10. (15 pts) Find the eigenvectors for each eigenvalue. Call them $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$.

a. $\lambda_1 = \boxed{}$:

$$\vec{v}_1 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

b. $\lambda_2 = \boxed{}$:

$$\vec{v}_2 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

c. $\lambda_3 = \square$:

$\vec{v}_3 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

11. (6 pts) Find the eigenpolynomials for each eigenvalue. Call them q_1, q_2 and q_3.
Verify they are eigenpolynomials by checking that $L(q_k) = \lambda_k q_k$ using the definition of L.

$q_1 = \square$ $L(q_1) =$

$q_2 = \square$ $L(q_2) =$

$q_3 = \square$ $L(q_3) =$

12. (6 pts) The eigenpolynomials $q = (q_1, q_2, q_3)$ form a second basis for P_3.
Find the matrix of the linear map L relative to the q basis. Call it $\underset{q \leftarrow q}{D}$.

$$\underset{q \leftarrow q}{D} = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

13. (15 pts) Find the change of basis matrices $\underset{e \leftarrow q}{C}$ and $\underset{q \leftarrow e}{C}$.

$$\underset{e \leftarrow q}{C} = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

$$\underset{q \leftarrow e}{C} = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

14. (5 pts) Verify your matrices satisfy $\underset{e \leftarrow q}{C} \ \underset{q \leftarrow q}{D} \ \underset{q \leftarrow e}{C} = \underset{e \leftarrow e}{A}$.

15. (5 pts) Compute $\left(\underset{e \leftarrow e}{A}\right)^5$.