1. Consider the vector space \( W = \text{Span}(1, \sin^2 x, \cos^2 x, \sin x \cos x) \) with the usual addition and scalar multiplication of functions. Do the following 4 vectors 
\[ f_1 = 1, \quad f_2 = \sin^2 x, \quad f_3 = \cos^2 x, \quad f_4 = \sin x \cos x \]
form a basis? If yes, prove it. If no, pare it down to a basis and prove it is a basis.

2. Consider the vector space \( V = \text{Span}(1, e^x, e^{-x}) \) with the usual addition and scalar multiplication of functions.

   a. Show \( e_1 = 1, \quad e_2 = e^x \) and \( e_3 = e^{-x} \) are a basis for \( V \). What is the dimension of \( V \)?
      HINT: Since they already span \( V \), all you need to show is linear independence.

   b. Show \( E_1 = 1, \quad E_2 = \sinh x = \frac{e^x - e^{-x}}{2} \) and \( E_3 = \cosh x = \frac{e^x + e^{-x}}{2} \) are another basis for \( V \).
      HINT: Why do you only need to show one of spanning or linear independence?

   c. Find \( C_e^E \), the change of basis matrix from the \( E \)-basis to the \( e \)-basis.

      NOTE: If the bases are taken as rows:
      \[
      e = (e_1, e_2, e_3) = (1, e^x, e^{-x}) \quad \text{and} \quad E = (E_1, E_2, E_3) = (1, \sinh x, \cosh x)
      \]
      and the components of a vector \( \vec{v} \) are columns \((\vec{v})_e\) and \((\vec{v})_E\) satisfying \( \vec{v} = e(\vec{v})_e = E(\vec{v})_E \),
      then this matrix satisfies: \((\vec{v})_e = C(e) (\vec{v})_E\).

   d. Find \( C_e^E \), the change of basis matrix from the \( e \)-basis to the \( E \)-basis.

   e. For the function \( q = 7 + 4 \sinh x - 2 \cosh x \), find the components relative to the \( E \)-basis.
      Then use \( C_e^E \) to find the components relative to the \( e \)-basis.
      Then check your work by substituting \( \sinh x \) and \( \cosh x \) directly into the function.

   f. For the function \( r = 5 - 2e^x + 4e^{-x} \), find the components relative to the \( e \)-basis.
      Then use \( C_e^E \) to find the components of \( r \) relative to the \( E \)-basis.
      Then check your work by substituting \( \sinh x \) and \( \cosh x \) into the answer.