1. Consider the vector space $W = \text{Span}(1, \sin^2 x, \cos^2 x, \sin x \cos x)$ with the usual addition and scalar multiplication of functions. Do the following 4 vectors

$$\vec{f}_1 = 1, \quad \vec{f}_2 = \sin^2 x, \quad \vec{f}_3 = \cos^2 x, \quad \vec{f}_4 = \sin x \cos x$$

form a basis? If yes, prove it. If no, pare it down to a basis and prove it is a basis.

2. Consider the vector space $V = \text{Span}(1, e^x, e^{-x})$ with the usual addition and scalar multiplication of functions.

a. Show $\vec{e}_1 = 1$, $\vec{e}_2 = e^x$ and $\vec{e}_3 = e^{-x}$ are a basis for V. What is the dimension of V?
HINT: Since they already span V, all you need to show is linear independence.

b. Show $\vec{E}_1 = 1$, $\vec{E}_2 = \sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\vec{E}_3 = \cosh x = \dfrac{e^x + e^{-x}}{2}$ are another basis for V.
HINT: Why do you only need to show one of spanning or linear independence?

c. Find $\underset{e \leftarrow E}{C}$, the change of basis matrix from the E-basis to the e-basis.
NOTE: If the bases are taken as rows:

$$e = (\vec{e}_1, \vec{e}_2, \vec{e}_3) = (1, e^x, e^{-x}) \quad \text{and} \quad E = \left(\vec{E}_1, \vec{E}_2, \vec{E}_3\right) = (1, \sinh x, \cosh x)$$

and the components of a vector $\vec{v}$ are columns $(\vec{v})_e$ and $(\vec{v})_E$ satisfying $\vec{v} = e\,(\vec{v})_e = E\,(\vec{v})_E$
then this matrix satisfies: $(\vec{v})_e = \underset{e \leftarrow E}{C}\,(\vec{v})_E$ and $E = e\,\underset{e \leftarrow E}{C}$.

d. Find $\underset{E \leftarrow e}{C}$, the change of basis matrix from the e-basis to the E-basis.

e. For the function $q = 7 + 4\sinh x - 2\cosh x$, find the components relative to the E-basis.
Then use $\underset{e \leftarrow E}{C}$ to find the components relative to the e-basis.
Then check your work by substituting $\sinh x$ and $\cosh x$ directly into the function.

f. For the function $r = 5 - 2e^x + 4e^{-x}$, find the components relative to the e-basis.
Then use $\underset{E \leftarrow e}{C}$ to find the components of r relative to the E-basis.
Then check your work by substituting $\sinh x$ and $\cosh x$ into the answer.