1. A plate has the shape of the region between the curves

\[ y = 1 + \frac{1}{2} e^x, \quad y = 2 + \frac{1}{2} e^x; \]
\[ y = 3 - \frac{1}{2} e^x, \quad y = 5 - \frac{1}{2} e^x \]

where \( x \) and \( y \) are measured in centimeters.

If the mass density is \( \rho = ye^x \) \( \text{gm/cm}^2 \), find the total mass of the plate.

HINT: Use the curvilinear coordinates \((u,v)\) defined by

\[ y = u + \frac{1}{2} e^x \quad \text{and} \quad y = v - \frac{1}{2} e^x \]

Follow these steps:

a. Write the boundaries in terms of \( u \) and \( v \).

b. Define the curvilinear coordinate system \((x,y) = \vec{R}(u,v)\) by solving for \( x \) and \( y \) in terms of \( u \) and \( v \).

c. Compute the Jacobian factor.

d. Express the mass density in terms of \( u \) and \( v \).

e. Compute the mass.
2. Verify Stokes’ Theorem \( \iint_P \nabla \times \vec{F} \cdot d\vec{S} = \oint_{\partial P} \vec{F} \cdot d\vec{r} \) for the vector field \( \vec{F} = (-yz, xz, z^2) \) and the paraboloid \( z = x^2 + y^2 \) with \( z \leq 4 \) oriented down and out. Follow these steps:

**LHS:**
- a. Compute \( \nabla \times \vec{F} \).

- b. Parametrize the paraboloid by \( \vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, r^2) \) and compute the normal vector, \( \vec{N} \).

- c. Evaluate \( \nabla \times \vec{F} \) on the surface.

- d. Compute \( \nabla \times \vec{F} \cdot \vec{N} \).

- e. Compute the surface integral.

**RHS:**
- f. Parametrize the boundary curve, \( \vec{r}(\theta) \), and compute the tangent vector, \( \vec{v}(\theta) \).

- g. Evaluate \( \vec{F} \) on the curve.

- h. Compute \( \vec{F} \cdot \vec{v} \).

- i. Compute the line integral.
3. Verify Gauss’ Theorem \[ \iiint_{H} \nabla \cdot \vec{F} \, dV = \iint_{\partial H} \vec{F} \cdot d\vec{S} \] for the vector field \( \vec{F} = (xz, yz, x^2 + y^2) \) and the solid hemisphere, \( H \), given by \( x^2 + y^2 + z^2 \leq 4 \) with \( z \geq 0 \). Be careful with orientations. Follow these steps:

**LHS:**

a. Compute \( \nabla \cdot \vec{F} \).

b. Compute the volume integral.

**RHS:** The boundary of the hemisphere consists of the hemispherical surface, \( S \), given by \( x^2 + y^2 + z^2 = 9 \) with \( z \geq 0 \) and the disk, \( D \), given by \( x^2 + y^2 \leq 9 \) with \( z = 0 \).

c. Parametrize the hemispherical surface, \( S \), by \( \vec{R}(\phi, \theta) = (2 \sin \phi \cos \theta, 2 \sin \phi \sin \theta, 2 \cos \phi) \) and compute the normal vector, \( \vec{N} \).

d. Evaluate \( \vec{F} = (xz, yz, x^2 + y^2) \) on the hemispherical surface.

e. Compute \( \vec{F} \cdot \vec{N} \).

f. Compute the surface integral \( \iint_{S} \vec{F} \cdot d\vec{S} \) on the hemispherical surface.

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g. Parametrize the disk, \( D \), by \( \mathbf{R}(r, \theta) = (r \cos \theta, r \sin \theta, 0) \) and compute the normal vector, \( \mathbf{N} \).

h. Evaluate \( \mathbf{F} = (xz, yz, x^2 + y^2) \) on the disk.

i. Compute \( \mathbf{F} \cdot \mathbf{N} \).

j. Compute the surface integral \( \iint_D \mathbf{F} \cdot d\mathbf{S} \) on the disk.

k. Compute the total surface integral \( \iint_{\partial H} \mathbf{F} \cdot d\mathbf{S} \).