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MATH 311 Section 501 Quiz on Integration Spring 2013 P. Yasskin

1. A plate has the shape of the region between the curves

$$y = 1 + \frac{1}{2}e^x, \qquad y = 2 + \frac{1}{2}e^x,$$

$$y = 3 - \frac{1}{2}e^x, \qquad y = 5 - \frac{1}{2}e^x$$

where x and y are measured in centimeters.

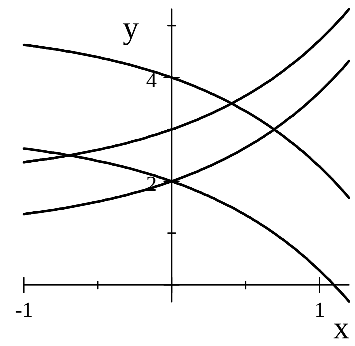
If the mass density is $\rho = ye^x$ gm/cm^2,

find the total mass of the plate.

HINT: Use the curvilinear coordinates (u,v) defined by

$$y = u + \frac{1}{2}e^x \quad \text{and} \quad y = v - \frac{1}{2}e^x$$

Follow these steps:



a. Write the boundaries in terms of u and v.

b. Define the curvilinear coordinate system $(x,y) = \vec{R}(u,v)$ by solving for x and y in terms of u and v.

c. Compute the Jacobian factor.

d. Express the mass density in terms of u and v.

e. Compute the mass.

2. Verify Stokes' Theorem $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial P} \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (-yz, xz, z^2)$ and the paraboloid $z = x^2 + y^2$ with $z \le 4$ oriented down and out. Follow these steps:

LHS:

a. Compute $\vec{\nabla} \times \vec{F}$.

b. Parametrize the paraboloid by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$ and compute the normal vector, $\vec{N}$.

c. Evaluate $\vec{\nabla} \times \vec{F}$ on the surface.

d. Compute $\vec{\nabla} \times \vec{F} \cdot \vec{N}$.

e. Compute the surface integral.

RHS:

f. Parametrize the boundary curve, $\vec{r}(\theta)$, and compute the tangent vector, $\vec{v}(\theta)$.

g. Evaluate $\vec{F}$ on the curve.

h. Compute $\vec{F} \cdot \vec{v}$.

i. Compute the line integral.

3. Verify Gauss' Theorem $\iiint_H \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial H} \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (xz, yz, x^2 + y^2)$ and the solid hemisphere, H, given by $x^2 + y^2 + z^2 \leq 4$ with $z \geq 0$. Be careful with orientations. Follow these steps:

LHS:

a. Compute $\vec{\nabla} \cdot \vec{F}$.

b. Compute the volume integral.

RHS: The boundary of the hemisphere consists of the hemispherical surface, S, given by $x^2 + y^2 + z^2 = 9$ with $z \geq 0$ and the disk, D, given by $x^2 + y^2 \leq 9$ with $z = 0$.

c. Parametrize the hemispherical surface, S, by $\vec{R}(\varphi, \theta) = (2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi)$ and compute the normal vector, $\vec{N}$.

d. Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the hemispherical surface.

e. Compute $\vec{F} \cdot \vec{N}$.

f. Compute the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ on the hemispherical surface.

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g. Parametrize the disk, D, by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\varphi, 0)$ and compute the normal vector, $\vec{N}$.

h. Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the disk.

i. Compute $\vec{F} \cdot \vec{N}$.

j. Compute the surface integral $\iint_D \vec{F} \cdot d\vec{S}$ on the disk.

k. Compute the total surface integral $\iint_{\partial H} \vec{F} \cdot d\vec{S}$.