Name_____ ID_____

MATH 311 Exam 1 Spring 2001

Section 200 P. Yasskin

1	/15	5	/10
2	/30	6	/10
3	/ 5	7	/10
4	/10	8	/10

1. (15 points) Consider the three points

$$P = (1,2,3) \qquad Q = (1,3,4) \qquad R = (2,3,3)$$

a. (5 pts) Find parametric equations of the plane containing P, Q and R.

b. (5 pts) Find a non-parametric equation of the plane containing P, Q and R.

c. (5 pts) Find an equation of the line through P perpendicular to the plane containing P, Q and R.

2. (30 points) Consider the equations

$$\begin{aligned} 2x + 4y + 6z &= 4 \\ v + y &= 2 \\ w + z &= b \\ 2v + w + x + 4y + 4z &= 3 \end{aligned}$$

a. (5 pts) Write out the augmented matrix for this system.

b. (10 pts) For what value(s) of b do there exist solutions?

c. (10 pts) For those value(s) of b, find all solutions.

d. (5 pts) Circle the geometric description of the solution set:

point, line, 2-plane, 3-plane, 4-plane, $\mathbf{R}^5$

3. (5 points) In $\mathbf{R}^5$ with coordinates (v, w, x, y, z), write out an equation of the 3-plane through the point $P = (5, 4, 3, 2, 1)$ with tangent vectors

$$\vec{a} = (2, 1, 3, 0, 4) \qquad \vec{b} = (1, -1, 2, -2, 3) \qquad \vec{c} = (2, 1, 3, -1, 0)$$

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} =$$

4. (10 points) Duke Skywater is flying the Millennium Eagle along the curve

$$\vec{r}(t) = (2\cos t, 3\sin t, t)$$

At $t = \frac{\pi}{2}$, he releases a garbage pod which travels along his tangent line with constant velocity (equal to his velocity at the time of release). Where is the garbage pod at $t = \pi$?

5. (10 points) Duke Skywater is flying the Millennium Eagle through the galactic polaron field. At $t = 20$, Duke's position is $\vec{r} = (20, 10, 30)$ lightyears and his velocity is $\vec{v} = (.1, .3, .2)$ $\frac{\text{lightyears}}{\text{year}}$. At that time, he measures the density of polarons to be $\rho = 15 \times 10^6$ $\frac{\text{polarons}}{\text{lightyear}^3}$ and the gradient of this density to be $\vec{\nabla}\rho = (2 \times 10^6, -1 \times 10^6, 3 \times 10^6)$ $\frac{\text{polarons}}{\text{lightyear}^4}$.

a. What does he measure as the time rate of change the polaron density, $\frac{d\rho}{dt}$?

b. Using a linear approximation, what would he expect the polaron density to be at the point $\vec{x} = (21, 12, 29)$?

6. (10 points) Duke Skywater is flying the Millennium Eagle through the galactic hyperon field. At $t = 20$, Duke's position is $\vec{r} = (20, 10, 30)$ lightyears and his velocity is $\vec{v} = (.1, .3, .2)$ $\frac{\text{lightyears}}{\text{year}}$. At that time, he measures the hyperon field and its Jacobian to be

$$\vec{H} = \begin{pmatrix} 200 \\ 150 \\ 300 \end{pmatrix} \text{Hans} \qquad D\vec{H} = \begin{pmatrix} 30 & -10 & 20 \\ -40 & 10 & 5 \\ -10 & 0 & 10 \end{pmatrix} \frac{\text{Hans}}{\text{lightyear}}$$

a. What does he measure as the time rate of change the hyperon field, $\frac{d\vec{H}}{dt}$?

b. Using a linear approximation, what would he expect the hyperon field to be at $t = 22$?

7. (10 points) Consider the vector space $\mathbf{R}^+$ of all positive real numbers with the operations of

Vector Addition: $x \oplus y = xy$ (real number addition)

Scalar Multiplication: $\alpha \odot x = x^{\alpha}$ (real number exponentiation)

Translate each of the following statements into ordinary arithmetic.

a. For all x we have $0 \odot x = \vec{0}$.

b. For all a we have $a \odot \vec{0} = \vec{0}$.

c. If $a \odot x = \vec{0}$ then either $a = 0$ or $x = \vec{0}$.

8. (10 points) Consider the linear function $L : R^3 \longrightarrow R^2$ given by

$$L(\vec{u}) = \begin{pmatrix} \int_0^1 (u_1 + u_2 x + u_3 x^2)\, dx \\ \frac{d}{dx}(u_1 + u_2 x + u_3 x^2)\Big|_{x=1} \end{pmatrix}$$

Find the matrix A of the linear function, so that you can rewrite it as

$$L(\vec{u}) = A\vec{u}$$