Name__________________________ ID_____________

MATH 311 Exam 3 Spring 2001

Section 200 P. Yasskin

1	/20	3	/30
2	/10	4	/40

1. $\left(20 \text{ points}\right)$ Let V be the vector space of functions spanned by $v_1 = t$ and $v_2 = t^2$ with the inner product

$$\langle f,g\rangle = \int_0^1 f(t)\,g(t)\,dt$$

a. (10) Find $\cos\theta$ where θ is the angle between v_1 and v_2.

b. (10) Apply the Gram-Schmidt procedure to $\{v_1,v_2\}$ to produce an orthonormal basis $\{u_1,u_2\}$.

2. $(10$ points$)$ In $\mathbf{R}^5$ find the volume of the parallepiped with edges

$$\vec{a} = (1,0,2,0,1)$$
$$\vec{b} = (0,2,1,0,-1)$$
$$\vec{c} = (-1,0,0,2,1)$$

3. $\left(30\text{ points}\right)$ A paraboloid in $\mathbf{R}^4$ with coordinates (w,x,y,z), may be parametrized by

$$(w,x,y,z) = \vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2, r^2)$$

for $0 \le r \le \sqrt{3}$ and $0 \le \theta \le 2\pi$.

a. (10) Find the area of the surface.

b. (10) Compute $P = \iint \sqrt{1+8w^2+8x^2}\, dS$ over the surface.

c. (10) Compute $I = \iint (xy\,dw\,dz - wz\,dx\,dy)$ over the surface.

$$w = r\cos\theta, \qquad x = r\sin\theta, \qquad y = r^2, \qquad z = r^2$$

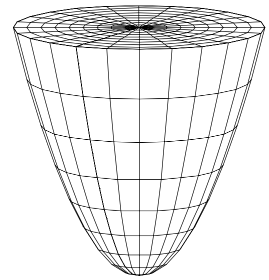
4. (40 points) The solid paraboloid V at the right is given by $x^2 + y^2 \le z \le 4$.

It's boundary (denoted by ∂V) has two parts:

The paraboloid P given by $z = x^2 + y^2$ for $z \le 4$.

The disk D given by $x^2 + y^2 \le 4$ with $z = 4$.

Let $\vec{G} = (xz^2, yz^2, z(x^2 + y^2))$.



a. (5) Compute $\vec{\nabla} \cdot \vec{G}$.

b. (10) Compute $\displaystyle\iiint_V \vec{\nabla} \cdot \vec{G}\, dV$ over the solid paraboloid V.

HINT: Use cylindrical coordinates.

c. (15) Compute $\iint_{P\downarrow} \vec{G} \cdot d\vec{S}$ over the paraboloid P with normal pointing DOWN and OUT.

HINT: Parametrize the paraboloid with coordinates (r,θ).

d. (5) Compute $\iint_{D\uparrow} \vec{G} \cdot d\vec{S}$ over the disk D with normal pointing UP.

HINT: Parametrize the disk with coordinates (r,θ).

e. (5) Compute $\iint_{\partial V} \vec{G} \cdot d\vec{S} = \iint_{P\downarrow} \vec{G} \cdot d\vec{S} + \iint_{D\uparrow} \vec{G} \cdot d\vec{S}$

(Note: By Gauss' Theorem, the answers to (b) and (e) should be equal.)