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1	/30	4	/20
2	/10	5	/10
3	/15	6	/15

MATH 311 Final Exam Spring 2001

Section 200 P. Yasskin

1. (30 points) Let $S(2,2)$ be the set of 2×2 symmetric matrices, i.e. 2×2 matrices M satisfying $M^{\mathsf{T}} = M$. Consider the function $L : M(2,2) \to S(2,2)$ given by $L(X) = X + X^{\mathsf{T}}$.

a. (5) Show that $S(2,2)$ is a subspace of $M(2,2)$, the vector space of 2×2 matrices.

b. (5) Find a basis for $S(2,2)$. What is the dimension of $S(2,2)$?

c. (5) Show L is linear.

d. (15) For the linear function L, identify

- (1) $Dom(L) =$

  $\dim Dom(L) =$

- (1) $CoDom(L) =$

  $\dim CoDom(L) =$

- (3) $Ker(L) =$

  $\dim Ker(L) =$

- (3) $Ran(L) =$

  $\dim Ran(L) =$

- (3) $1-1$? Circle: Yes No
  Why?

- (3) $onto$? Circle: Yes No
  Why?

- (1) Verify the Nullity-Rank Theorem for L.

2. $(10$ points$)$ Consider the function of two matrices $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $Y = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ given by

$$\langle X, Y \rangle = tr(XY)$$

where tr means "trace" which is the sum of the principle diagonal entries, i.e. $tr\begin{pmatrix} w & x \\ y & z \end{pmatrix} = w + z$.

Explain why $\langle\ ,\ \rangle$ is an inner product on $S(2,2)$, but is not an inner product on $M(2,2)$.

3. (15 points) Consider the linear map $L : P_2 \to \mathbf{R}^3$ given by $L(p) = \begin{pmatrix} p(-1) \\ p(0) \\ p(1) \end{pmatrix}$.

a. (5) Find the matrix of L relative to the bases $e = \{e_1 = 1, e_2 = t, e_3 = t^2\}$ for P_2 and $i = \left\{ \vec{i}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{i}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{i}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ for $\mathbf{R}^3$. Call it A.

b. (5) Find the matrix of L relative to the bases $q = \{q_1 = 1 + t^2, q_2 = t + t^2, q_3 = t^2\}$ for P_2 and $v = \left\{ \vec{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}$ for $\mathbf{R}^3$. Call it B.

c. (5) Find the matrix B by a second method.
($\Leftarrow$ Use opposite page.)

4. (20 points) Consider the helix H parametrized by $\vec{r}(t) = (4\cos t, 4\sin t, 3t)$ between $A = (4,0,0)$ and $B = (-4,0,3\pi)$.

a. (10) Compute the line integral $\int_{A \atop H}^{B} \vec{F} \cdot d\vec{s}$ of the vector field $\vec{F} = (yz, -xz, z)$ along the helix H.

b. (10) Find the total mass of the helix H if the linear mass density is $\rho = z^2$.

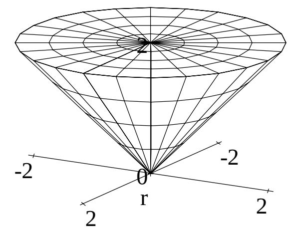
5. $(10$ points$)$ Compute $\oint x\ dx + z\ dy - y\ dz$ around the boundary of the triangle with vertices $(0,0,0)$, $(0,1,0)$ and $(0,0,1)$, traversed in this order of the vertices.

HINT: The yz-plane may be parametrized as $\vec{R}(u,v) = (0,u,v)$.

6. (15 points) Gauss' Theorem states

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$$

where ∂V is the total boundary of V with OUTWARD normal.



Let V be the solid cone $\sqrt{x^2+y^2} \le z \le 2$.

Let C be the conical surface $z = \sqrt{x^2+y^2}$ for $z \le 2$
with UPWARD normal.

Let D be the disk $x^2+y^2 \le 4$ with $z = 2$
with UPWARD normal.

Compute $\iint_C \vec{F} \cdot d\vec{S}$ for $\vec{F} = (xy^2, yx^2, z^3)$ in two ways.

a. (5) Method I: Parametrize C and compute $\iint_C \vec{F} \cdot d\vec{S}$ explicitly.

b. (10) Method II: Parametrize D and V, compute $\iint_D \vec{F} \cdot d\vec{S}$ and $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$ and solve for $\iint_C \vec{F} \cdot d\vec{S}$. Be very careful with the orientation of the surfaces.