

Sample problems for Test 1

Any problem may be altered or replaced by a different one!

Problem 1 (20 pts.) Find the point of intersection of the planes $x + 2y - z = 1$, $x - 3y = -5$, and $2x + y + z = 0$ in \mathbb{R}^3 .

Problem 2 (30 pts.) Let $A = \begin{pmatrix} 1 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 2 & 0 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}$.

- (i) Evaluate the determinant of the matrix A .
- (ii) Find the inverse matrix A^{-1} .

Problem 3 (20 pts.) Let \mathcal{P}_4 be the vector space of all polynomials (with real coefficients) of degree less than 4. Determine which of the following subsets of \mathcal{P}_4 are vector subspaces. Briefly explain.

- (i) The set S_1 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0) = 0$.
- (ii) The set S_2 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0)p(1) = 0$.
- (iii) The set S_3 of polynomials $p(x) \in \mathcal{P}_4$ such that $(p(0))^2 + (p(1))^2 = 0$.
- (iv) The set S_4 of polynomials $p(x) \in \mathcal{P}_4$ such that $p(0) = 0$ and $p(1) = 1$.

Problem 4 (30 pts.) Let $B = \begin{pmatrix} 0 & -1 & 4 & 1 \\ 1 & 1 & 2 & -1 \\ -3 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$.

- (i) Find the rank and the nullity of the matrix B .
- (ii) Find a basis for the row space of B , then extend this basis to a basis for \mathbb{R}^4 .

Bonus Problem 5 (25 pts.) Show that the functions $f_1(x) = x$, $f_2(x) = xe^x$, and $f_3(x) = e^{-x}$ are linearly independent in the vector space $C^\infty(\mathbb{R})$.

Bonus Problem 6 (20 pts.) Let V and W be subspaces of the vector space \mathbb{R}^n such that $V \cup W$ is also a subspace of \mathbb{R}^n . Show that $V \subset W$ or $W \subset V$.