MATH 304–510 Spring 2017

# Suggested homework for Quiz 11

**Problem 1.** Consider a linear operator $K : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$K(\mathbf{x}) = C\mathbf{x}, \quad \text{where} \quad C = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}.$$

(i) Explain why $K$ is a rigid motion and, specifically, a rotation about an axis.
(ii) Find the axis of rotation.
(iii) Find the angle of rotation.

**Problem 2.** Let $L$ denote a linear operator on $\mathbb{R}^3$ that acts on vectors from the standard basis as follows: $L(\mathbf{e}_1) = \mathbf{e}_3$, $L(\mathbf{e}_2) = \mathbf{e}_1$, $L(\mathbf{e}_3) = \mathbf{e}_2$.

(i) Explain why $L$ is a rigid motion.
(ii) Is $L$ a rotation about an axis? Is $L$ a reflection in a plane? Explain your answers.
(iii) If $L$ is a rotation, find the axis and the angle. If $L$ is a reflection, find the plane. If $L$ is neither rotation nor reflection, describe the action of $L$ in geometric terms.

**Problem 3.** Find the matrix of the rotation by 180° about the line spanned by the vector $\mathbf{a} = (1, 1, 1)$.

**Problem 4.** Find the matrix of the reflection in the plane $x - y + z = 0$.

**Problem 5.** Let $R_1$ be the counterclockwise rotation of $\mathbb{R}^3$ about the $x$-axis by 90° and $R_2$ be the clockwise rotation of $\mathbb{R}^3$ about the $z$-axis by 90°. The composition $S = R_2 \circ R_1$ of these two transformations is also a rotation about an axis. Find the angle of the rotation $S$.