

## Sample problems for Test 1

Any problem may be altered or replaced by a different one!

**Problem 1** Find a quadratic polynomial  $p(x)$  such that  $p(1) = 1$ ,  $p(2) = 3$ , and  $p(3) = 7$ .

**Problem 2** Let  $A$  be a square matrix such that  $A^3 = O$ .

- (i) Prove that the matrix  $A$  is not invertible.
- (ii) Prove that the matrix  $A + I$  is invertible.

**Problem 3** Let  $A = \begin{pmatrix} 1 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 2 & 0 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}$ .

- (i) Evaluate the determinant of the matrix  $A$ .
- (ii) Find the inverse matrix  $A^{-1}$ .

**Problem 4** Determine which of the following subsets of  $\mathbb{R}^3$  are subspaces. Briefly explain.

- (i) The set  $S_1$  of vectors  $(x, y, z) \in \mathbb{R}^3$  such that  $xyz = 0$ .
- (ii) The set  $S_2$  of vectors  $(x, y, z) \in \mathbb{R}^3$  such that  $x + y + z = 0$ .
- (iii) The set  $S_3$  of vectors  $(x, y, z) \in \mathbb{R}^3$  such that  $y^2 + z^2 = 0$ .
- (iv) The set  $S_4$  of vectors  $(x, y, z) \in \mathbb{R}^3$  such that  $y^2 - z^2 = 0$ .

**Problem 5** Let  $B = \begin{pmatrix} 0 & -1 & 4 & 1 \\ 1 & 1 & 2 & -1 \\ -3 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$ .

- (i) Find the rank and the nullity of the matrix  $B$ .
- (ii) Find a basis for the row space of  $B$ , then extend this basis to a basis for  $\mathbb{R}^4$ .
- (iii) Find a basis for the nullspace of  $B$ .

**Problem 6** Show that the functions  $f_1(x) = x$ ,  $f_2(x) = xe^x$ , and  $f_3(x) = e^{-x}$  are linearly independent in the vector space  $C^\infty(\mathbb{R})$ .

**Problem 7** Let  $V$  be a finite-dimensional vector space and  $V_0$  be a proper subspace of  $V$  (where proper means that  $V_0 \neq V$ ). Prove that  $\dim V_0 < \dim V$ .