

# Homework assignment #1

## (due Friday, January 25)

All problems are from Williamson/Trotter.

Problem 1.2.3 means Chapter 1, Section 2, Exercise 3.

**1.1.9.** Find numbers  $a$  and  $b$  such that  $a\mathbf{x} + b\mathbf{y} = (9, -1, 10)$ , where  $\mathbf{x} = (3, -1, 0)$ ,  $\mathbf{y} = (0, 1, 5)$ . Is there more than one solution?

**1.1.13.** Let  $\mathbf{x} = \mathbf{i} + \mathbf{j}$ ,  $\mathbf{y} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ , and  $\mathbf{z} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ . Calculate (a)  $-\mathbf{x} + 2\mathbf{y} - \mathbf{z}$ , (b)  $6\mathbf{x} - 2\mathbf{y} + \mathbf{z}$ , and (c)  $-4\mathbf{x} + 3\mathbf{y} + \mathbf{z}$ .

**1.1.28.** A small factory produces products of four different kinds. The vectors  $\mathbf{w} = (50, 75, 100, 190)$  and  $\mathbf{r} = (100, 150, 200, 300)$  give the wholesale and retail prices in dollars for a single unit of each kind. The vector  $\mathbf{p} = (25, 25, 15, 10)$  gives the number of units of each product produced in a day.

- What vector gives the retailer's profit per unit for the four products?
- If the wholesale price vector is doubled, what happens to the retailer's profit vector?
- What happens to the retailer's profit vector if the retail prices each increase by 10% and the wholesale prices stay unchanged?
- What vector gives the total production for a five-day week?

**1.2.25.** Verify that  $|\left((1-t)\mathbf{x} + t\mathbf{y}\right) - \mathbf{x}| = t|\mathbf{y} - \mathbf{x}|$  and  $|\left((1-t)\mathbf{x} + t\mathbf{y}\right) - \mathbf{y}| = (1-t)|\mathbf{y} - \mathbf{x}|$  for  $\mathbf{x}, \mathbf{y}$  in  $\mathbb{R}^n$  and  $0 \leq t \leq 1$ . Why is the condition  $0 \leq t \leq 1$  needed?

**1.2.28.** Draw the triangle with vertices at  $(-2, 0)$ ,  $(3, 1)$ , and  $(0, 4)$ . Mark arrowheads on the three sides so that following the arrows takes you around the triangle in the counterclockwise direction. Write down the vectors represented by the three arrows and calculate their sum.

**1.2.29.** Can you state a general condition on a set of arrows that implies that the vectors they represent add up to  $\mathbf{0}$ ? (The condition should apply to the previous problem.) Does the condition work in  $\mathbb{R}^3$  as well as in the plane?

**1.3.5.** Let  $\mathbf{a} = (-1, 1)$ ,  $\mathbf{b} = (0, 1)$ ,  $\mathbf{c} = (2, 1)$ , and  $\mathbf{d} = (-3, 2)$ .

- Sketch the lines  $t\mathbf{a} + \mathbf{b}$  and  $s\mathbf{c} + \mathbf{d}$ .
- Find the point  $\mathbf{p}$  where the lines in part (a) intersect by finding values of  $s$  and  $t$  for which  $\mathbf{p} = t\mathbf{a} + \mathbf{b} = s\mathbf{c} + \mathbf{d}$ .
- Change  $\mathbf{c}$  to  $(2, -2)$  and show that then the lines  $t\mathbf{a} + \mathbf{b}$  and  $s\mathbf{c} + \mathbf{d}$  do not intersect.

**1.3.17.** Sketch the plane passing through the three points  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 1)$ . Find a representation for this plane in the parametric form  $t_1\mathbf{u}_1 + t_2\mathbf{u}_2 + \mathbf{v}$ .

**1.3.19.** A plane  $P$  in  $\mathbb{R}^3$  contains the line  $t(1, -1, 2) + (1, 2, 1)$  and the point  $(3, 0, 1)$ .

- Find a vector between two points of  $P$  that is linearly independent of  $(1, -1, 2)$ .
- Find a parametric representation for  $P$ .

**1.4.7.** Let  $\mathbf{u} = (2, 1, 2)$  and  $\mathbf{v} = (1, 2, 2)$ . Find (a)  $\mathbf{u} \cdot \mathbf{v}$ , (b)  $|\mathbf{u}|$  and  $|\mathbf{v}|$ , and (c) the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

**1.4.15.** Let  $\mathbf{x} = (2, -3, 1)$  and  $\mathbf{v} = (1, 3, -2)$ . Find (a) the coordinate and (b) the component of the vector  $\mathbf{x}$  in the direction of the vector  $\mathbf{v}$ , and also (c) the component of  $\mathbf{x}$  perpendicular to  $\mathbf{v}$ .

**1.4.18.** For the triangle with the vertices  $A = (1, 2, 4)$ ,  $B = (-2, -1, 2)$ , and  $C = (4, 2, -3)$ , find the lengths of its sides, and determine which of its angles are acute, obtuse, or right angles.

**1.5.14.** Find the cosine of the angle between the plane  $2x + y + z = 1$  and a line parallel to  $(1, 2, 1)$ .

**1.5.19.** Find an equation for the plane parallel to the plane  $3x - 2y + 5z = 2$  and passing through  $(2, 1, 1)$ .

**1.5.27.** Find the distance from the point  $(1, 0, -1)$  to the plane  $x + 2y + 3z = 1$ . Is the point on the same side of the plane as the origin or on the opposite side? Is the point above the plane or below it (where “up” is the direction of the positive  $z$ -axis)?