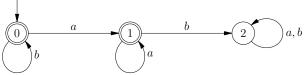
MATH 433 Spring 2015

## Sample problems for Exam 2

Any problem may be altered, removed or replaced by a different one!

**Problem 1.** Let R be a relation defined on the set of positive integers by xRy if and only if  $gcd(x, y) \neq 1$  ("is not coprime with"). Is this relation reflexive? Symmetric? Transitive?

**Problem 2.** A Moore diagram below depicts a 3-state acceptor automaton over the alphabet  $\{a, b\}$  which accepts those input words that do not contain a subword ab (and rejects any input word containing a subword ab). Prove that no 2-state automaton can perform the same task.



**Problem 3.** List all cycles of length 3 in the symmetric group S(4). Make sure there are no repetitions in your list.

**Problem 4.** Write the permutation  $\pi = (4\ 5\ 6)(3\ 4\ 5)(1\ 2\ 3)$  as a product of disjoint cycles.

**Problem 5.** Find the order and the sign of the permutation  $\sigma = (1\ 2)(3\ 4\ 5\ 6)(1\ 2\ 3\ 4)(5\ 6)$ .

**Problem 6.** What is the largest possible order of an element of the alternating group A(10)?

**Problem 7.** Consider the operation \* defined on the set  $\mathbb{Z}$  of integers by a\*b=a+b-2. Does this operation provide the integers with a group structure?

**Problem 8.** Let M be the set of all  $2\times 2$  matrices of the form  $\binom{n-k}{0-n}$ , where n and k are rational numbers. Under the operations of matrix addition and multiplication, does this set form a ring? Does M form a field?

**Problem 9.** Let L be the set of the following  $2\times 2$  matrices with entries from the field  $\mathbb{Z}_2$ :

$$A = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \\ \begin{bmatrix} 0 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \end{pmatrix}, \quad B = \begin{pmatrix} \begin{bmatrix} 1 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \\ \begin{bmatrix} 0 \end{bmatrix} & \begin{bmatrix} 1 \end{bmatrix} \end{pmatrix}, \quad C = \begin{pmatrix} \begin{bmatrix} 1 \end{bmatrix} & \begin{bmatrix} 1 \end{bmatrix} \\ \begin{bmatrix} 1 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \end{pmatrix}, \quad D = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & \begin{bmatrix} 1 \end{bmatrix} \\ \begin{bmatrix} 1 \end{bmatrix} & \begin{bmatrix} 1 \end{bmatrix} \end{pmatrix}.$$

Under the operations of matrix addition and multiplication, does this set form a ring? Does L form a field?

**Problem 10.** For any  $\lambda \in \mathbb{Q}$  and any  $v \in \mathbb{Z}$  let  $\lambda \odot v = \lambda v$  if  $\lambda v$  is an integer and  $\lambda \odot v = v$  otherwise. Does this "selective scaling" make the additive Abelian group  $\mathbb{Z}$  into a vector space over the field  $\mathbb{Q}$ ?