Homework assignment #3
(due Friday, September 22)

Problem 1. For the following functions, sketch the Fourier series of $f(x)$ (on the interval $-L \leq x \leq L$) and determine the Fourier coefficients:

(i) $f(x) = e^{-x}$;  
(ii) $f(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases}$

Problem 2. For the following functions, sketch $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$:

(i) $f(x) = 1$;
(ii) $f(x) = 1 + x$;
(iii) $f(x) = \begin{cases} x, & x < 0, \\ 1 + x, & x \geq 0; \end{cases}$
(iv) $f(x) = e^x$;
(v) $f(x) = \begin{cases} 2, & x \leq 0, \\ e^{-x}, & x > 0. \end{cases}$

Problem 3. Sketch the Fourier sine series of $f(x) = x$. Also, roughly sketch the sum of a (large) finite number of nonzero terms of the Fourier sine series.

Problem 4. (i) Consider a function $f(x)$ which is even around $x = L/2$. Show that the odd coefficients ($n$ odd) of the Fourier cosine series of $f(x)$ on $0 \leq x \leq L$ are zero.

(ii) Explain the result of part (i) by considering a Fourier cosine series of $f(x)$ on the interval $0 \leq x \leq L/2$.

Problem 5. Fourier series can be defined on other intervals besides $-L \leq x \leq L$. Suppose that a smooth function $g(y)$ is defined for $a \leq y \leq b$. Represent $g(y)$ using periodic trigonometric functions with period $b - a$. Determine formulas for the coefficients.

*Hint: use the linear transformation* $y = \frac{a + b}{2} + \frac{b - a}{2L}x$. 