Problem 1. Solve Laplace’s equation inside a rectangle \(0 \leq x \leq L, 0 \leq y \leq H\), with the following boundary conditions:

\[
\begin{align*}
\frac{\partial u}{\partial x}(0, y) &= 0, \\
\frac{\partial u}{\partial x}(L, y) &= 0, \\
u(x, 0) &= 0, \\
u(x, H) &= f(x).
\end{align*}
\]

Problem 2. Solve Laplace’s equation inside a semicircle of radius \(a\) \((0 < r < a, 0 < \theta < \pi)\) subject to the boundary conditions: \(u = 0\) on the diameter and \(u(a, \theta) = g(\theta)\).

Problem 3. Solve Laplace’s equation inside a 90° sector of a circular annulus \((a < r < b, 0 < \theta < \pi/2)\) subject to the boundary conditions:

\[
\begin{align*}
u(r, 0) &= 0, \\
u(r, \pi/2) &= 0, \\
u(a, \theta) &= 0, \\
u(b, \theta) &= f(\theta).
\end{align*}
\]

Problem 4. Consider the heat equation in a two-dimensional rectangular region, \(0 < x < L, 0 < y < H\),

\[
\frac{\partial u}{\partial t} = k \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

subject to the initial condition \(u(x, y, 0) = f(x, y)\).

Solve the initial-boundary value problem and analyze the temperature as \(t \to \infty\) if the boundary conditions are:

\[
\begin{align*}
\frac{\partial u}{\partial x}(0, y, t) &= 0, \\
\frac{\partial u}{\partial x}(L, y, t) &= 0, \\
\frac{\partial u}{\partial y}(x, 0, t) &= 0, \\
\frac{\partial u}{\partial y}(x, H, t) &= 0.
\end{align*}
\]

Problem 5. Consider the wave equation for a vibrating rectangular membrane \((0 < x < L, 0 < y < H)\)

\[
\frac{\partial^2 u}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

subject to the initial conditions \(u(x, y, 0) = 0\) and \(\frac{\partial u}{\partial t}(x, y, 0) = f(x, y)\).

Solve the initial-boundary value problem if

\[
\begin{align*}
\frac{\partial u}{\partial x}(0, y, t) &= 0, \\
\frac{\partial u}{\partial x}(L, y, t) &= 0, \\
\frac{\partial u}{\partial y}(x, 0, t) &= 0, \\
\frac{\partial u}{\partial y}(x, H, t) &= 0.
\end{align*}
\]