

6. Direction field (sections 1.1)

1. Previously we studied several classes of first order differential equations that can be explicitly solved. However, there is no method to solve general first order equation

$$y' = F(t, y) \quad (1)$$

Instead, one tries to study how solutions look like without actually solving the equation. One way to do it is using the following geometric interpretation of a solution of (1):

A function $y = y(t)$ is a solution of the ODE (1) if and only if for every t the tangent line to the graph of $y(t)$, has slope $F(t, y(t))$.

This motivates the following construction: for each point (t, y) consider a straight line passing through this point and having the slope $F(t, y)$.

The collection of all such lines forms the *direction field* of equation (1) (this is by analogy with the notion of a vector field you learned in Calculus 3 or in Physics classes, when for every point a vector starting at this point is chosen: the difference here is that for every point a straight line, passing through this point, is chosen).

Imagine that the direction field is drawn on the floor (with t -axis and y -axis are given) and you start at some point (the initial condition) and try to move along the graph of a solution of (1), i.e. along an integral curve of (1). The direction field prescribes the direction of your motion at every time moment.

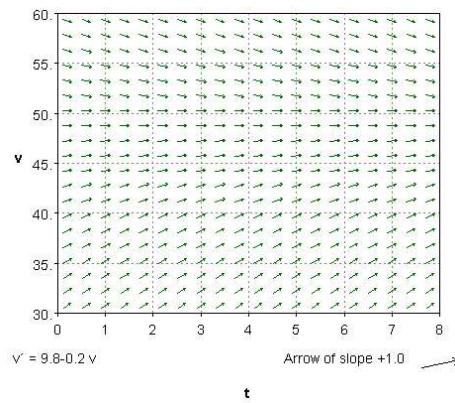
2. To sketch direction field use the following steps:
 - Choose a rectangular grid of points in the ty -plane.
 - Calculate the slopes of tangent lines to the integral curves at the gridpoints.
 - Draw a short line segment of the tangent lines through the gridpoints.

The direction field drawn on a sufficiently dense grid gives quite accurate picture of how the graphs of solution (the integral curves) look like.

3. **FALLING HAILSTONE** Consider hailstone with mass $m = 0.03kg$ and drag coefficient $\gamma = 0.006kg/s$
 - (a) Write down the ODE describing the motion of the hailstone.

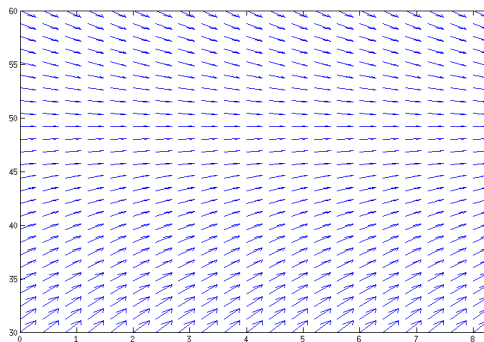
- (b) Sketch the direction field

- (c) The recommended software to draw direction fields is Java script **dfield** that can be downloaded freely from John C. Polking's web-page, Rice University (the link is provided on my course webpage, all installation instructions are there, you need Java installed on your PC to run this). Here are the result of using this script for the equation in item 3 a. See another option to draw this using MatLab in the next item.



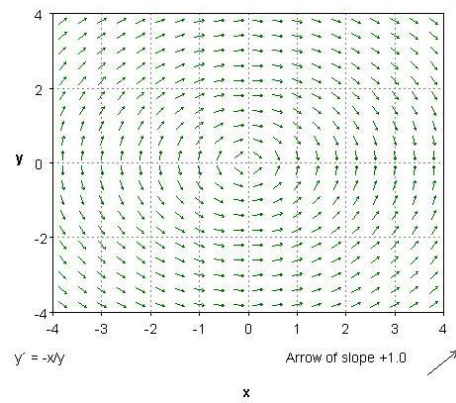
- (d) We can use Matlab to get directional field (see section 6.1 of the recommended book on MatLab for more detail):

```
[t,v]=meshgrid(0:0.4:8, 30:1.2:60);
S=9.8-0.2*v;
quiver(t,v,ones(size(S)),S), axis tight
```



4. Given the equation $y' = -x/y$

- (a) Sketch the direction field for this equation in the region containing the origin. Use the same scale/range for x and y variables;



(b) Guess what are the integral curves of this equation from this sketch

(c) Confirm your guess in the previous item by actually solving this equation.