Homework Assignment #1 FALL 2017 - MATH308

due September 6, 2017 at the beginning of class

Topics covered : equations $y' = ay + b$, where a and b are constant, and separable equations (corresponds to sections 1.2, 2.2 in the textbook), method of integrating factor (sections 2.1), and a bonus question on equation that can be reduced to separable by an appropriate substitution (based on Enrichment 1 lecture notes). *You do not need to use calculator for this assignment.*

1. Assume that the velocity v of the falling object satisfies the following differential equation:

$$v'(t) = 9.8 - \frac{v(t)}{a} \tag{1}$$

where a is a positive constant.

 (a) Find a number v_e such that $v(t) \equiv v_e$ is a solution of equation (1) (in other words find the equilibrium solution of (1)).

 (b) Solve the equation (1) with initial condition $v(0) = 0$. What is the limit of this solution when $t \to +\infty$? How this limiting velocity is related to your answer in the item (a)?

 (c) Find the time that must elapse for the object to reach 25% of the limiting velocity found in the item (b).

 (d) How far does the object fall in the time found in the item (c).

2. Solve the following differential equations (find the general solutions):

 (a) $y' = (\cos t)\, y + \cos t$;

 (b) $(xy + 2y)dy - (y^2 + 4)dx = 0$.

3. Find the general solution of the differential equation

$$y' + 3y = 5e^{-3t},$$

and determine how the solutions behave as $t \to +\infty$.

4. Solve the initial value problem

$$y' = \frac{3y}{t} + t^5 \cos t, \quad y(\pi) = 4.$$

5. (*bonus* - 30 points) Before attempting this problem review the enrichment 1 lecture notes from week 1, where I discuss the equation of the type $y' = f(\frac{y}{x})$ (so-called, homogeneous equations) and $y' = f(ax+by+c)$: the main idea here is to make an appropriate substitution to obtain a separable equation: $u(x) = \frac{y(x)}{x}$ in the first case and $u(x) = ax + by(x) + c$ in the second case. Then find the general solution of the following equations:

 (a) $y' = \dfrac{5x - 3y}{3x - y}$;

 (b) $y' = (x + 3y - 4)^2$.

Note that it is not sufficient just to reduce the equation to a separable one as done in the examples in the enrichment notes. You need also to solve the obtained separable equation and then to return to the original function $y(x)$