We will discuss the description of classes of contact equivalent ordinary differential equations of order $n \geq 5$ in terms of Cartan bundle $B$ with the base being the space of solutions and with the structure group, which is the semidirect product of $GL(2, \mathbb{R})$ with $\mathbb{R}^n$. In case of ODEs of order 5 we give necessary and sufficient conditions for contact equivalent classes of ODEs to define a Cartan connection on $B$ with values in the Lie algebra of $GL(2, \mathbb{R})$ semidirect product of $\mathbb{R}^5$. Relations between this geometry and recently developed $SO(3)$ geometry in dimension 5 will also be discussed.