Homework Assignment 3 in Topology I, MATH636
due to Sept 21, 2009
(all problems except problem 7 will be graded, but you are encouraged to solve problem 7 as well)

1. Solve problem 4, p. 10 in the text.

2. Prove the following form of the intermediate value theorem. Let $X$ be a connected space and let $f : X \to \mathbb{R}$ be a map into $\mathbb{R}$ with the Euclidean topology. Show that if $a = f(x_0)$ and $b = f(x_1)$ with $a < b$, for some $x_0, x_1 \in X$, then the interval $(a, b)$ is contained in the image of $X$.

3. a. Show that if $A$ is dense in the topological space $X$ and $A$ is connected, then $X$ is connected.
   
b. Let $X$ be a connected topological space and $A$ be a closed set in $X$. Show that if the boundary $\partial A$ of $A$ is connected, then $A$ is connected. Is it necessary true, if $X$ is not connected?

4. a. Describe the components of $\mathbb{R}_\ell$ (the lower limit topology).
   
b. Let $I = [0, 1]$. Describe the components and arc components of $I_{\text{lex}}^2$ (the lexicographic topology)

5. A topological space $X$ is a door space if every subset is either open or closed. Show that a Hausdorff door space has at most one accumulation point, and if $x$ is a point which is not an accumulation point, then $\{x\}$ is open. Give an example of a non-discrete door space.
