Trigonometric integral formulas
By Z. Teitler for Math 152 at Texas A&M 2009

$$\int \sin^m x \cos^n x \, dx$$

- Odd cosine (n odd): substitute $u = \sin x$, $du = \cos x \, dx$
  - save one $\cos x$ (it is going to be used in du)
  - change rest of $\cos x$'s to $\sin x$'s using $\cos^2 x = 1 - \sin^2 x$

- Odd sine (m odd): substitute $u = \cos x$, $du = -\sin x \, dx$
  - save one $\sin x$
  - change rest of $\sin x$'s to $\cos x$'s using $\sin^2 x = 1 - \cos^2 x$

- Both even: Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.
  Now, powers are lower. Go back to the start (does sine or cosine have an odd power?).

$$\int \tan^m x \sec^n x \, dx$$

- Even secant (n even): substitute $u = \tan x$, $du = \sec^2 x \, dx$
  - save $\sec^2 x$
  - change rest of $\sec x$'s to $\tan x$'s using $\sec^2 x = 1 + \tan^2 x$

- Odd tangent (m odd): substitute $u = \sec x$, $du = \sec x \tan x \, dx$
  - save $\sec x \tan x$
  - change rest of $\tan x$'s to $\sec x$'s using $\tan^2 x = \sec^2 x - 1$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \sin Sx \cos Cx \, dx = \int \frac{1}{2}[\sin((S-C)x) + \sin((S+C)x)] \, dx$$

$$\int \sin Ax \sin Bx \, dx = \int \frac{1}{2}[\cos((A-B)x) - \cos((A+B)x)] \, dx$$

$$\int \cos Ax \cos Bx \, dx = \int \frac{1}{2}[\cos((A-B)x) + \cos((A+B)x)] \, dx$$

- For the first one, S is the number in Sine and C is the number in Cosine.
- For the next two, switch the order if necessary so that the bigger number is first (in A) and the smaller number is second (in B). For example, in $\int \sin 6x \sin 3x \, dx$ they are already in the right order; in $\int \sin 4x \sin 9x \, dx$ you should switch it to $\int \sin 9x \sin 4x \, dx$.
- (You don't have to switch them. It is just so that $A - B$ won't be negative.)