Week in Review XII

courtesy of David J. Manuel

Section 6.4
1. State and prove the Fundamental Theorem of Calculus, part 1.

2. State and prove the Fundamental Theorem of Calculus, part 2.

3. The graph of \( f \) is shown below. If \( g(x) = \int_0^x f(t) \, dt \), determine when the graph of \( g \) is:
   a) increasing
   b) concave down.

Section 6.5
4. Define \( \ln(x) = \int_1^x \frac{1}{t} \, dt \). Using this definition, prove:
   a) \( \ln(xy) = \ln x + \ln y \)
   b) \( \ln \left( \frac{x}{y} \right) = \ln x - \ln y \)

5. Prove: If \( f \) is an integrable, even function, then \( \int_{-a}^{a} f(x) \, dx = 2 \int_0^{a} f(x) \, dx \).

6. Evaluate \( \int \sin x \cos x \, dx \) using 2 different substitutions. Explain why you get different answers.