1. Ancient Geometry. Euclidean geometry from the classical viewpoint of an axiom bases system.

- Analyse the classical axioms of Euclid in comparison of the modern axioms as formulated by Hilbert. What important features, or hidden assumptions, did the Greeks make that were essentially undiscovered for two millenia? What role did non-euclidean geometry play in uncovering the flaws?
- Describe in some detail the contents of Book X, wherein the method of exhaustion is developed? In the proposition relating the areas and diameters of two circles, discuss the proximity of Euclid’s argument to a modern limiting process. Show how the method of exhaustion could not be used in place of the modern limit.
- Describe the three great problems of antiquity. How were they solved using non-standard curves? Trace the mathematics developed (e.g. algebraic numbers, surds) in the service of solving these problems.

2. Infinity.

- What are inaccessible infinities? Trace their history. What possible uses do they have?
- Give a careful analysis of ancient views of infinity, particularly those of Aristotle. Which of the ancients attempted to break from this view? And how?
- The paradoxes of infinity have caused many difficulties in the early days of set theory. What are they and how were they resolved? Is there universal acceptance of the resolutions. Categorize them.
- Outline the medieval conception of infinity. What role did the scholastic model play? What arguments were used to undo it?
- What is constructivism? Regarding infinity, what role did the constructivists, led say by Kronecker, play? What role do constructivists play today?
- Who is Kurt Gödel? What was his impact on infinity and set theory and mathematics in general? How did his work impact Hilbert’s program?

3. Number theory from the Greeks to the 20th century.

- Describe in detail the number theory found in Euclid? What was the connection to the Pythagoreans? What was understood about proportion and geometric progressions?
- Trace the importance of symbols to the development of modern number theory. For example, how would Fermat’s Little Theorem have been formulated and proved, à la Euclid?
- Fermat’s Last Theorem has involved many mathematicians and resulted in the creation of much new mathematics. Fill in the details of this thesis.
- Discuss modern data encryption historically, in terms of the development of the theory and construction of prime numbers.
• What is elementary number theory? What is analytic number theory? What is algebraic number theory? Compare the three. Trace their historical roots. Give typical results.

4. Calculus — from Archimedes to Euler to Riemann to Lebesgue

• Explain why all the interesting results of Apollonius and Archimedes, though prodigious, cannot be classified as calculus today. What limitations were a consequence of Greek philosophical views? Carefully describe your arguments in terms of Archimedes’ quadratures and Apollonius’s tangencies. What was the flaw in Apollonius’ normals, and how was this flaw replicated by Descartes, though in a more sophisticated context.

• Newton and Leibnitz both contributed to the “invention” of calculus. What does this mean? Who was first? What controversies arose? What was the ultimate impact on British analysis?

• Explain the roles of Cauchy and Riemann in bringing the integral to its modern form. Describe in detail the role of integral first as area, then as antiderivative, then as area again. What flaws in their work were finally resolved by Lebesgue? What importance contributions came from Weierstrass?

5. The emergence of algebra

• What role did the roots of a polynomial play in the creation of group structures. Who were the main contributors. How did Cauchy use his “permutations?”

• Trace the history of the Fundamental Theorem of Algebra from its formulation by Girard through its first correct proof by Gauss. What major mistakes were made? Analyse the various proofs available today.

6. Diophantine analysis.

• Show how the Fermat Last Theorem fits into Diophantine analysis? What sort of techniques are available toward solving these Diophantine-type problems. Here you may want to include an introduction to and history of elliptic curves.

• Give an account of Hilbert’s 10th problem. Who is Julia Robinson?

• Discuss modern applications of Diophantine-type problems.