Chapter 5 Notes by Joe Kahlig
Fall 2004

Section 5.1 Simple and Compound Interest

• Simple interest is when interest is computed on the original principal only.
  – The two formulas used for these problems are: \( I = Prt \) and \( A = P(1+rt) \).
    
    \( I = \text{Interest earned or paid} \)
    
    \( r = \text{simple interest rate} \)
    
    \( t = \text{time (usually in years)} \)
    
    \( A = \text{Accumulated amount of the account} \)
    
    – Example: Find the balance of an account if $500 is deposited for 5 months at the simple interest rate of 7% per year.
    
    Answer: \( A = 500 \left( 1 + \frac{.07 \times 5}{12} \right) = $514.58 \)

• Compound interest is when interest is computed on the original principal and also on the interest.
  – The two formulas (actually only one) associated with this type of problem are:
    
    Accumulated amount: \( A = P(1 + i)^n \) \( \text{or} \) \( A = P \left( 1 + \frac{r}{m} \right)^{mt} \)
    
    The present value formula is just solving the above equation for the variable \( P \).
    
    Present value: \( P = \frac{A}{(1 + i)^n} \) \( \text{or} \) \( P = A(1 + i)^{-n} \)
    
    also: \( P = A \left( 1 + \frac{r}{m} \right)^{-mt} \)
    
    \( A = \text{Accumulated amount of the account} \)
    
    \( P = \text{Principal (start of the account)} \)
    
    \( r = \text{Nominal interest rate per year} \)
    
    \( m = \text{Number of compounding periods in a year} \)
    
    \( t = \text{Time measured in years} \)
    
    \( n = mt, \text{which is the total number of periods.} \)
    
    \( i = \frac{r}{m}, \text{which is the interest rate per period.} \)
    
    – Example: Bob purchased a 5 year $15,000 promissory note with an interest rate of 6% per year compounded monthly. How much should Bob pay for the note?
    
    – Example: How much will be in the account at the end of 3 years if $800 is invested at 6%/year compounded
      * annually
      * semiannually
      * quarterly
      * monthly
      * daily
• The effective yield (annual yield) is a way to compare accounts with different interest rates and compounding styles. This number can be computed with the formula, \( r_e = 100 \left( 1 + \frac{r}{m} \right)^m - 100 \), or by a calculator command, \( \text{Eff} \). To get to the finance menu: on the TI-83 press [2nd] [x^-1] or on the TI-83 plus or TI-84 press [APPS] and select the Finance... choice. Go down the list and select the \( \text{Eff} \) command. The syntax for this command is \( \text{Eff}(r, m) \). Thus, \( \text{Eff}(8, 12) \) will compute the effective yield on an account with a nominal rate of 8% compounded monthly.

**Sections 5.2 and 5.3 Annuities.**

• An Annuity is an account where fixed payments are made in a regular manner. example: car loan, house loan, saving for college,...

• All of these types of problems can be solved using the TVM solver that is built into the TI-83. If you are using the TI-83 press [2nd] [x^-1] and then press [ENTER]. If you are using the TI-83 plus, the TI-84, or the TI-84 plus press the [APPS] and the select the Finance application and press enter. Here are the variables that are used in the TVM Solver.

\[
\begin{align*}
N &= m \cdot t \text{ which is the total number of periods(compoundings) for the life of the account.} \\
I\% &= \text{The interest rate per year as a percentage.} \\
PV &= \text{The present value(starting value) of the account.} \\
PMT &= \text{This is the payment that is made each period.} \\
FV &= \text{The future value(end value) of the account.} \\
P/Y &= \text{The number of payments per year.} \\
C/Y &= \text{The number of compoundings per year.}
\end{align*}
\]

For this class, \( P/Y \) and \( C/Y \) are equal and \( \text{PMT:END BEGIN} \) should be set to END.

Example: Suppose we have an savings account that has an interest rate of 3% per year compounded quarterly. We decide to invest $1000 for 5 years. How much money is in the account at the end of the five year period?

Enter the following into the variables of the TVM solver.

\[
\begin{align*}
N &= 5 \cdot 4 \\
I\% &= 3 \\
PV &= -1000 \quad \text{The negative tells which direction the money is flowing. If money goes away} \\
\text{form you, then make it negative.} \\
PMT &= 0 \quad \text{Since the $1000 is a single deposit, not a recurring deposit.} \\
P/Y &= 4 \\
C/Y &= 4
\end{align*}
\]

To solve for \( FV \), move the cursor to the \( FV \) line and press [ALPHA] [ENTER].

The answer to this problem is $1161.18.
Class problems over chapter 5.

1. Bob wants to deposit $100 at the end of every six months into an IRA that pays 7% compounded semiannually. How much money is in the account at the end of 25 years? How much money did Bob deposit into the account? How much interest did Bob get on the account?

2. Sue is starting her bank account with $1000 and will deposit $100 each month for the next 6 years. If the account pays 5.25% interest compounded monthly, How much will she have in the account at the end of 6 years? How much interest did she earn?

3. You have decided to set up a college fund for your kid. You have decided that $75,000 should be enough to get Jr most of the way through college. You decide to open the account when Jr is born and make monthly deposits every month for 18 years. What is your monthly deposit that would reach your goal, if the account pays 4.75% interest compounded monthly? How much money did you deposit into the account? How much interest did the account earn?

4. You have decided that you want a new Chevy Truck in 4 years. It will cost $30,000. What monthly payments should you make into an account, paying 5% interest compounded monthly, so that you can buy the truck with cash?

5. After you retire, you would like to receive quarterly payments of $5,000 for the next 15 years. The account earns 8% compounded quarterly. What amount do you use to start the account. How much interest was earn during the lifetime of the account?

6. You just won $4 million in the lottery, but you choose the payment method instead of the cash option. This means that you will receive 20 payments of $200,000. How much should be deposited into an account paying 9% compounded annually, so that you can receive the your payments for the next 19 years?
7. You have decided that you want a new Chevy Truck. However you want it right now. It will cost $32,000. What monthly payments should you make, paying 4.5% interest compounded monthly, so that you can payoff the truck in 5 years?

8. You want to buy a house. The house will cost $120,000. You put 5% down and then plan to pay off the house for the next 30 years with monthly payments. What payments should you make to pay off the loan if the interest rate is 7.5% compounded monthly? How much interest did you pay?

9. repeat the house loan for a 15 year loan.

10. Write an amortization schedule for the first 4 months for problem 8.

<table>
<thead>
<tr>
<th>period</th>
<th>interest owed</th>
<th>payment</th>
<th>amt. toward principal</th>
<th>outstanding principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

11. How much equity would you have on the 30 year note after 5 years? after 10 years?
   \[ \text{Equity} = \text{value of the object} - \text{what you still owe} \]

12. You had made payments on the 30 year mortgage for 7 years and then decide to refinance the loan for 15 years at 5.35%. What would be the new house payment? How much money would you save, not counting refinancing costs, by refinancing the loan?

On the web page, under the Handout section, there are some additional problems for chapter 5. I would suggest looking at them.