Ballistics Project

M442, Fall 2003

1 Overview

Ballistics is the science of projectile motion and impact, phenomena well described by Newtonian mechanics. The number of applications of this type of analysis is staggering, ranging from such mundane issues as automobile accident simulations and optimal golfing to the critical studies of missile defense and space exploration. Somewhat less dramatically, in this project we will use Newtonian mechanics to describe the flight of a sponge dart, light enough so that air resistance will play a critical role.

As this is the first project of the semester, it will also serve as our introduction to MATLAB and LyX (see course handouts), two software packages that we will use extensively throughout the semester. The project is divided into weekly assignments that will not be due, but that should give you an idea of how far along you should be to avoid a painful final week. The hard deadline for this project—typed report and all—is Friday, October 10, and projects will not be accepted after this. The numbered items in each section should be included in your final report. Working ahead is, of course, encouraged, though you should be warned that many of the techniques you will need for carrying out your analysis will be discussed in class.

2 Experimental data

The data for this project was collected by firing sponge darts from a toy gun ($3.99, Wal-Mart—it’s not all that easy talking the department into financing this sort of thing). First, I tabulated a set of measurements for distance traveled (by the projectile) versus angle of inclination of the gun, taking angles of inclination 5, 10, 15, ..., 85 degrees. The darts were fired from a height of .18 meters.

<table>
<thead>
<tr>
<th>Angle of inclination</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance traveled</td>
<td>4.37</td>
<td>5.23</td>
<td>6.95</td>
<td>7.84</td>
<td>8.17</td>
<td>8.69</td>
<td>8.81</td>
<td>8.99</td>
<td>8.95</td>
</tr>
</tbody>
</table>

Next, I computed the time it took for a dart fired straight up from a height of .39 meters to hit the ground: 2.13 seconds. Finally, I computed the time it took one of the darts to fall 4.06 meters: .95 seconds. These fairly simple measurements will suffice for the assignments in this project.

3 Week 1: Analysis in the absence of air resistance

Ignoring air resistance, Newton’s equations of motion for an object under the influence of gravity alone are quite straightforward. Letting \( y \) represent the height of the object and \( x \) represent its distance traveled, we have\(^3\)

\[^3\text{We're also ignoring the fact that the gravitational pull on an object above the earth depends upon its height above the earth. We take } g = 9.81 \text{m/s}^2.\]
\[
\frac{d^2y}{dt^2} = -g, \quad \frac{d^2x}{dt^2} = 0. 
\]

1. Begin by fitting your data to a polynomial and determining the error of your fit, calculated as

\[
E = \sqrt{\sum_{k=1}^{17} (x_{m,k} - x_{e,k})^2},
\]

where \(x_{m,k}\) represents the distance according to your model and \(x_{e,k}\) represents the experimental distance.

2. Use the experimental data from Section 2 to determine the initial velocity with which the darts are fired.

3. Solve these two second order differential equations for \(x(t)\) and \(y(t)\) and use MATLAB to plot the \((x, y)\) trajectory obtained for angle of inclination \(\theta = 35^\circ\). Observe that \(x(0)\) and \(y(0)\) were determined by the set-up of my measurements and are known to be 0 and .18 m respectively. We will see in class that the remaining two initial conditions \(x'(0)\) and \(y'(0)\) are given by

\[
x'(0) = v \cos \theta, \quad y'(0) = v \sin \theta,
\]

where \(v\) is the velocity with which the gun fires its darts.

4. Determine the distance, \(d(\theta)\), your object travels as a function of \(\theta\), and compute the angle that maximizes its distance. Plot this function on MATLAB and compare it with my experimental values. Discuss the discrepancies.

5. Compute your error \(E\) for this model.

4 Week 2: Linear air resistance

A typical first attempt at introducing the effects of air resistance into a physical model is through linear air resistance, terms of the form \(-bv'\). Adding such terms to (1), we obtain

\[
\frac{d^2y}{dt^2} = -g - \frac{d}{dt}y(t), \quad \frac{d^2x}{dt^2} = -\frac{d}{dt}x(t),
\]

with the same initial conditions as before. We will now work through steps similar to those of Section 3, except that as the mathematics becomes more cumbersome, we will begin to fall back more on MATLAB.

1. Develop an analytic solution to equations (2).

2. Use your solution to equations (2) and the experimental data from Section 2 to determine the value of \(b\) for this model.

3. Find a (corrected) initial velocity \(v\).

4. Plot this trajectory (\(y\) versus \(x\)) for angle \(\theta = 35^\circ\) along with your similar trajectory in the absence of air resistance.

5. Analytically, determine an implicit equation for \(d(\theta)\), the distance the dart travels given that it was shot with angle of inclination \(\theta\). Determine the value of \(\theta\) that maximizes \(d(\theta)\).

6. Plot the three different versions of \(d(\theta)\)—experimental data, no air resistance data, and linear air resistance data—on the same figure, and again discuss the discrepancies.

7. Compute your error \(E\) for this model.
5 Weeks 3 and 4: Nonlinear air resistance

In general, linear air resistance is not an adequate model of projectile motion. In this final section of the project, you will work through the steps of the previous sections with a more physical form of air resistance. Unfortunately, analytic solutions for your model will be too cumbersome to derive or work with, so you will have to do most of your analysis with MATLAB.

1. Use the method of dimensional analysis to determine a more physical form for your air resistance.
2. Use MATLAB and the experimental data from Section 1 to determine the value of $b$ for your model.
3. Use your new value for $b$ to determine a (further corrected?) initial velocity $v$.
4. Use MATLAB’s event location to plot a trajectory for angle $\theta = 35^\circ$, and plot it along with your two previous trajectories at this angle.
5. Plot $d(\theta)$ for this model along with $d(\theta)$ from the previous sections. Determine the value of $\theta$ that maximizes $d(\theta)$ for this model.
6. Compute your error $E$ for this model and compare it with your errors from the previous models.

6 Week 5: Write-up

Most technical papers and reports end with a section summarizing the results and suggesting analyses that probably should have been included, but for various reasons—time constraints, budgetary constraints, sheer laziness—weren’t. For the Ballistics Project, your study was fairly exhaustive, so you will only need a summary, discussing which model worked best and how far off the others were. Any observations you make along the way should be discussed and interpreted. For example, during Weeks 3 and 4 you will determine the form of force due to air resistance. Once you get this, you should discuss its physical significance.