March, 1991
Updated September, 2010

Syllabus
Qualifying Examination
Complex Analysis

1. ARITHMETIC, GEOMETRY, AND TOPOLOGY OF THE COMPLEX NUMBERS: Field operations; stereographic projection; spherical metric; simple and multiple connectivity.

2. ANALYTIC FUNCTIONS: Cauchy–Riemann equations; power series; harmonic functions.

3. COMPLEX INTEGRATION: Cauchy's theorem; Goursat's proof; Cauchy's integral formula; residue theorem; computation of definite integrals by residues.

4. CONFORMAL MAPPING: linear fractional transformations and cross ratio; mappings by elementary functions; Riemann mapping theorem.

5. SINGULARITIES: classification of isolated singularities; Laurent series; Casorati–Weierstrass theorem; Picard's theorems.

6. GEOMETRIC FUNCTION THEORY: winding numbers and the argument principle; open mapping theorem; maximum principle; Schwarz lemma; three-circles theorem.

7. ANALYTIC CONTINUATION: Schwarz reflection principle; continuation along a path; monodromy theorem.

8. CONVERGENCE AND APPROXIMATION: normal families; Hurwitz's theorem; Runge's theorem; Mittag-Leffler's theorem; infinite products; factorization theorems of Weierstrass and Hadamard.

References:

- Lars V. Ahlfors, *Complex Analysis*, third edition, McGraw-Hill, 1979.
- Ralph P. Boas, *Invitation to Complex Analysis*, second edition, revised by Harold P. Boas, Mathematical Association of America, 2010.
- Robert B. Burckel, *An Introduction to Classical Complex Analysis*, Volume 1, Academic Press, 1979.
- John B. Conway, *Functions of One Complex Variable*, second edition, Springer-Verlag, 1978.
- Robert E. Greene and Steven G. Krantz, *Function Theory of One Complex Variable*, third edition, American Mathematical Society, 2006.